Go Groups and Polyominos

Polyominos are generalizations of dominos. A domino is a plane with two squares that share a side. A triomino is a plane with three squares, each two of which share a side.

Polyominos have a number of different possible arrangements. Symmetric, rotated, reflected, rotated and reflected, or translated patterns are the same as the pattern and so do not count.

A unique pattern has a unique boundary or surface. Knowing the surface defines the region, and vice versa.

Note: Geometrically, points connected diagonally can transform to points connected vertically or horizontally. For example, V can be L rotated 45 degrees.

Note: Cellular automata make only polyominos. Cellular automata transform polyominos to other polyominos.

In the game of GO, stones of one color can form a connected group. All points must connect horizontally or vertically. One stone is equivalent to one square of a polyomino, so GO groups have the same number of arrangements as polyominos. A GO-playing computer could store GO groups and their surrounding stones.

One Color

The empty polyomino has no square and has one arrangement. The single-square polyomino has one square and one arrangement. The domino has two squares and one arrangement. The triomino has three squares and two arrangements.

Therefore, the number of arrangements for polyominos of 2 to 28 squares is:
1, 2, 5, 12, 35, 108, 369, 1285, 4655, 17073, 63600, 238591, 901971, 3426576, 13079255, 50107909 or 50107911, 192622052, 742624232, 2870671950, 11123060678, 43191857688, 168047007728, 654999700403, 2557227044764, 9999088822075, 39153010938487, 153511100594603.

For n = 3,   2 = 1*2.
For n = 4,   5 = 1*5.
For n = 5,   12 = 2*6 = 2*2*3.
For n = 6,   35 = 5*7.
For n = 7,   108 = 9*12 = 2*2*3*3*3.
For n = 8,   369 = 9*41 = 9*35 + 108/2.
For n = 9, 1285 = 5*257 = 10*108 + 5*41.
For n = 10, 4655 = 5*7*7*19 = 7*19*35 = 11*369 + 11*54.
For n = 11, 17073 = 3*5691 = 3*3*1897 = 3*3*7*271
= 12*1285 + 3*19*29 = 3*4655 + 2*2*7*111 = 35*369 + 2*3*3*7*11.

For n = 3,   2 = 3*1 - 1
For n = 4,   5 = 3*2 - 1

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For n =  5,    12 = 3*5 - 3
For n =  6,    35 = 3*12 - 1
For n =  7,   108 = 3*35 + 3
For n =  8,   369 = 3*108 + 45
For n =  9,  1285 = 3*369 + 178
For n = 10,  4655 = 3*1285 + 800
For n = 11, 17073 = 3*4655 + 3108
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The differences are:

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1 1 2 5 12  35  108   369   1285    4655     17073
 0 1 3 7  23  73   261   916    3370    12418
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The ratios are:

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For n =  2,     1/1       = 1.00
For n =  3,     2/1       = 2.00
For n =  4,     5/2       = 2.50
For n =  5,    12/5       = 2.40
For n =  6,    35/12      = 2.92
For n =  7,   108/35      = 3.09
For n =  8,   369/108     = 3.41
For n =  9,  1285/369     = 3.48
For n = 10,  4655/1285    = 3.62
For n = 11, 17073/4655    = 3.67
For n = 12, 63600/17073   = 3.73
For n = 13, 238591/63600  = 3.75
For n = 14, 901971/238591 = 3.78
For n = 15, 3426576/901971 = 3.80
For n = 16, 13079255/3426576 = 3.82
For n = 17, 50107909/13079255 = 3.83
For n = 18, 192622052/50107909 = 3.84
For n = 19, 742624232/192622052 = 3.86
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Two Colors

With two different square colors allowed, the number of arrangements for polyominos with 1 to 5 squares is:
2, 3, 12, 54, 296.

Three Colors

With three different square colors allowed, the number of arrangements for polyominos with 1 to 4 squares is:
3, 6, 36, 246.

Four Colors

With four different square colors allowed, the number of arrangements for polyominos with 1 to 4 squares is:
4, 10, 80, 746.

Stone Number or Square Number

For one stone or square, as color number increases from 1 to 4, the sequence is:

1, 2, 3, 4.

For two stones or squares, as color number increases from 1 to 4, the sequence is:
1, 3, 6, 10.

For three stones or squares, as color number increases from 1 to 4, the sequence is:
2, 12, 36, 80.

For four stones or squares, as color number increases from 1 to 4, the sequence is:
5, 54, 246, 746.

GO Groups and Surrounds

GO groups with one stone can have 6 different surrounds.
GO groups with two stones can have 24 different surrounds.
GO groups with three stones can have 156 different surrounds.
The sequence is:
6, 24, 156.

For n > 1, this matches the sequence a(n)= 30 * a(n - 2) - a(n - 1).