Outline of Set Theory
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MATH>Set Theory

set theory

Set operations follow rules {set theory}. Boolean-set algebras {algebra of sets} can be axiomatic set theories. From sets, one can choose {selection, set} smaller sets {subset, selection}, without regard for sequence or order {combination, set}. For valid set-theory statements, if operations union and intersection interchange, null set and universal set interchange, and set inclusions reverse, valid statements result. Set inclusion can use only intersection and union, so statements do not need to have set inclusion.

set in mathematics

Objects {member, set} {element, set} and events can be in groups {set, mathematics}. Sets {well-defined set} can have membership criteria. Sets have numbers of members {cardinality, set} {power, set} {size, set}, which can be specific, unspecific, or infinite.

subsets

Sets can have parts {subset, set}. Sets can split into mutually exclusive subsets {partition, subset}. Subsets {connected set} can have all set members that are not in union of two separated sets.

union

Sets can contain all members of two sets {union, set}. Union of set with empty set is the set. Union of set with universal set is universal set. Union of relative complements is a set {symmetric difference}.

intersection

Sets can contain all members shared by two sets {intersection, set}. Intersection of set with empty set is empty set. Intersection of set with universal set is the set.

Venn diagram

Circles {Venn diagram} can be graphs that show set relations.

MATH>Set Theory>Set Types

universal set

Sets {universal set}| can contain everything.

empty set

Sets {null set} {empty set} can contain nothing.

countable set

Sets {countable set} can be in one-to-one correspondence with positive-integer set.

disjoint set

Two sets {disjoint set, non-overlapping} {non-overlapping set} can have no members in common.

equivalence class

Sets {equivalence class} can include all number examples. Equivalence classes are typically infinite sets. For example, the equivalence class of -2 can include all possible pairs of natural-number subtractions {equivalence relation, class}: -2 = 0 - 2 = (0,2) and -2 = 1 - 3 = (1,3), and so on. Complex numbers are equivalence classes of remainders of polynomials x / (x^2 + 1), where x is a real number.

residue class

All congruent integers form set {residue class, set}, [x]n or Zn.

MATH>Set Theory>Set Types>Interval

closed set

Sets {closed set, interval} can contain limit point at boundary.

derived set

Sets {derived set} can have other-set limit points.

open set

Sets {open set} can contain no boundary points and so have only interior points.

perfect set

All closed-set {perfect set} points can be at the limit.

MATH>Set Theory>Set Types>Ordered

monotonic set

Set systems {monotonic set} can have previous sets contained in next sets.

ordered set

Sets {ordered set} can have elements that follow trichotomy and transitivity relations. Sets {partially ordered set} can have elements that follow trichotomy and transitivity relations.

reflexive set

Relation between second coordinate and first coordinate can be same as relation between first coordinate and second coordinate {reflexive set}.

transitive set

Relation between second coordinate and first coordinate can be from higher to lower {transitive set}.

MATH>Set Theory>Set Types>Subset

complement of subset

Subsets {complement, set}| {absolute complement} can have all set members that do not belong to another subset. Subsets {relative complement} can have set members that do not belong to another subset and belong to third subset.

coset

Subsets {coset, subset} can contain products of one set element times all set elements.

nested set

Sets {nested set} {nest, set} can have one set inside the other.

power set

For a set, a set {power set} can contain all subsets. For n-element sets, power sets have 2^n elements. Subset number is greater than set-member number.

MATH>Set Theory>Axioms

Foundation axiom

Set theory does not allow sets {paradoxical set} that are elements of themselves {Foundation axiom}.

Zermelo-Fraenkel set theory

Set theories {Zermelo-Fraenkel set theory} {ZF set theory} can be axiomatic systems. Zermelo set theory has no paradoxes but is not consistent.

The empty set exists {axiom of empty set}. The empty set exists, so at least one set exists {axiom of existence}. Sets with same elements are equal {axiom of extension} {axiom of extensionality}. For two sets, another set exists that contains all and only elements of the two sets {axiom of union}. For two sets, another set exists that has the two sets as only elements {axiom of pairing}. For a set, another set exists whose elements are the subsets of the original set {axiom of powers} {axiom of power set}. A set exists that has the empty set as an element and, if an element is in the set, the set that contains only that element is an element in the set {axiom of infinity}.

Non-empty sets contain at least one element, and the non-empty set and the set of any element are disjoint sets {axiom of regularity}. For any set and any mapping, a subset of the set exists that has as elements the domain of the mapping {axiom of separation}. For any set and any mapping, a set exists that has as elements the range of the mapping over the original set's elements (as domain of mapping) {axiom of replacement} {axiom of specification}. For any set, a mapping exists that chooses one element of each subset (axiom of choice).

MATH>Set Theory>Axioms>Axiom Of Choice

axiom of choice

For any set, a mapping exists that chooses one element of each subset {axiom of choice} {Zermelo's axiom}. Elements are not in any other non-empty set, even if number of non-empty subsets is infinite. Axiom of choice is independent of set theory. Zermelo set theory has no paradoxes but is not consistent. If Zermelo-Fraenkel set theory is consistent without axiom of choice, then ZF set theory is consistent with axiom of choice.

Banach-Tarski theorem

Axiom of choice leads to unexpected consequences if applied to sets with uncountably infinite members. In such set, object can divide into five pieces that can rotate, translate, and invert to make much greater volume {Banach-Tarski theorem}.

Zorn lemma

In partially ordered sets in which subsets have upper bounds, sets have greatest member {Zorn's lemma} {Zorn lemma}. Zorn's lemma is equivalent to axiom of choice.

MATH>Set Theory>Infinite Set

infinite set

Sets {infinite set} can have unlimited numbers of elements, with no greatest element.

aleph naught

The smallest infinite set {aleph naught} is enumerable. Enumerable sets include cardinal numbers, ordinal numbers, integers, and rational numbers {algebraic-equation solutions}, which all have same size.

power of the continuum

The next-smallest infinite set is all points on a line {power of the continuum} {aleph one}, which is equivalent to all n-dimensional space points and to all real numbers. The second-smallest infinite set is all curves {aleph two}. Even-higher infinite sets exist, up to infinity {aleph infinity} {aleph naught naught}.

Cantor diagonal process

Rational-number sets have same infinity level as counting-number sets, as proved by Cantor's diagonal process {diagonalization} {Cantor diagonal process} {diagonal proof}.

rational numbers

Make table with infinite rows and columns. Cells have positive rational numbers. First row is a positive-integer series. Second row is a positive-integer series, each divided by 2. nth row is a positive-integer series, each divided by n.

To count fractions, start at top left 0. Go down one row to 0/2. Go diagonally up and right to first row at 1. Go right one column to 2. Go diagonally down and left to first column at 0/3. Repeat to cover all cells and count all fractions. Fractions count only once, establishing one-to-one correspondence between counting numbers and rational numbers.

real numbers

The real-number set has higher infinity than counting-number set. List real numbers in sequence as table. Rows are real numbers. Columns are digits. Along diagonals through table are real numbers, with one digit from each row and column. Change all digits of main-diagonal real number. Resulting real number is not any real number already in table, because all row and column digits have changed. Therefore, real numbers number more than counting numbers, and no one-to-one correspondence exists between counting numbers and real numbers.

continuum hypothesis

No set has size between integer-set size and real-number-set size {continuum hypothesis} {continuum problem}. No aleph is between aleph naught and aleph one. Raising any positive integer to aleph-naught power results in aleph one.

indeterminible

Continuum hypothesis is indeterminable under set theory.

generalized continuum hypothesis

Positive integers raised to aleph n power equal aleph n+1 {generalized continuum hypothesis}. Generalized continuum hypothesis is independent of set theory.

Cantor set

In continuous intervals, continually removing inner third of remaining continuous interval still leaves infinitely many points, and total empty distance is interval length {Cantor set}. Cantor sets are the same at all scales. 1/f noise is like Cantor sets.

Cantor paradox

Power sets are larger than their basis sets {Cantor's paradox} {Cantor paradox}. Therefore, there can be no largest set and no set of all sets.

Galileo paradox

Are infinite sets countable {paradox of Galileo} {Galileo paradox}?

MATH>Set Theory>Laws

absorption law

Union of set a and its intersection with another set b is the set {absorption law}: a V (a & b) = a. Intersection of set a and its union with another set b is the set: a & (a V b) = a.

idempotent law

Union of set a with itself is the set {idempotent law}: a V a = a. Intersection of set a with itself is the set: a & a = a.

De Morgan laws for sets

Absolute complement of set union equals intersection of set absolute complements {De Morgan's laws, set} {De Morgan laws, set} {De Morgan's complement}: ~(a V b) = ~a & ~b. Absolute complement of set intersection equals union of set absolute complements: ~(a & b) = ~a V ~b.

involution of set

Absolute complement of set absolute complement is set {involution, set}: ~(~a) = a. Absolute complement of empty set is universal set. Absolute complement of universal set is empty set.

MATH>Set Theory>Laws Of Form

laws of form

Sets have types {theory of types, laws of form} {laws of form}. Sets have lower or higher type than other sets. Lower sets cannot be in statements about higher sets, and higher sets cannot be in statements about lower sets.

differences

Laws of form use differences {calculus of indications}.

self-reference

In set type, boundary contains object surrounded, and object is boundary {self-reference, set}. Self-reference allows references to members, classes, classes of classes, and so on. Imaginary numbers express self-reference and self-referential statements, because imaginary numbers can represent time dimensions.

Statements that use set types correctly can use self-reference. Statements that do not use set types correctly must not use self-reference.

space or set

Members, definition, description, selection, name, or statement define space or set {space, laws of form}. Spaces can imply boundaries and have natural boundaries. Absolute time and relative time can be boundaries and show boundary history {creativity, laws of form}.

logical laws

Laws of form are equivalent to logical laws. NOT a = marked a. a OR b = NOT a AND NOT b. a AND b = NOT (NOT a OR NOT b). a THEN b = NOT a OR b.

boundary in sets

Sets have open or closed boundaries {boundary, set}. Actions can create a boundary {expansion, laws of form} or cross a boundary {contraction, laws of form}. Complex actions combine expansions and contractions.

marking

Boundary side can be positive {marked region} and other side negative {unmarked region}.

space

Drawing boundary inside space tells nothing about whole space. Encircling whole space with boundary tells nothing about whole space.

boundary making

Interaction between observer and system {boundary making} makes larger system containing both observer and original system. Observer can surround all or some sets or be inside a set in the set hierarchy.

Observer cannot know whole system, only part close to boundary. Observer makes boundaries to describe whole system.

interval

Regions {interval, laws of form} have boundaries. Boundaries can be in marked sets or spaces {open interval, set}. Boundaries can not be in marked sets or spaces {closed interval, set}.

operations at boundaries

Going from inside boundary to outside boundary is opposite operation {inverse, laws of form}. Going from inside boundary to outside boundary, and then going from inside boundary to outside boundary, results in original condition {identity, laws of form}. Drawing same boundary second time is NULL operation and gives no new information.

contraction statement

Two mutually exclusive same-type sets can become one same-type set, by crossing a boundary. Statements {contraction statement} can name two classes {categorizing, laws of form} and a relation {description, laws of form}, such as equality, with a reference point from which to relate members. Instructions {selection rule, laws of form} can be about how to select members. Processes can cross boundaries.

expansion statement

Sets can divide into two mutually exclusive same-type sets, by creating a boundary. Statements {expansion statement} can name {naming, laws of form} classes {definition, expansion} and a distinction, such as equality, with a reference point from which to identify members. Instructions {selection rule, expansion} can be about how to select members. Processes can draw boundaries.

MATH>Set Theory>Set Properties

abstraction principle

If all set members have a property, set has property {abstraction principle} {principle of abstraction}.

adjunction

Elements added to sets {adjunction} can extend set properties.

bound of set

Sets have a highest member {upper bound, set} {bound}. Sets have a lowest member {lower bound, set}. Sets can have minimum at highest member {greatest lower bound, set}. Sets can have maximum at lowest member {least upper bound, set}.

cyclic permutation

Ordered-set elements can advance one place {cyclic permutation}. Cyclic permutations can have only two elements {transposition, set}.

extension principle

Two sets are equal if and only if sets have same members {principle of extension} {extension principle}.

injection mapping

The identity relation can apply to domain subset {injection mapping}.

integral domain

Sets can have two binary operations {integral domain}. Operations make commutative rings and have identity elements. First operation has identity element if second operation happens.

interior point

Open sets can contain no point {interior point} at boundary.

one-to-one correspondence

Two sets are equivalent if their elements pair {one-to-one correspondence}|.

ordered in sets

Relations {ordered} between two or more things can have sequence. Sequence does not have to be complete {partial order}.

number

For two things, first {first coordinate} and second {second coordinate} make pair {ordered pair, set} or relation {binary relation}. First coordinate belongs to domain. Second coordinate belongs to range. Second coordinate derives from first coordinate {Cartesian product, pair}.

Relations can be among many things {ordered n-tuple} {n-ary relation}, with many coordinates.

types

Order can be linear {simple order} {linear order}. All set members can have simple order {chain, set}. Non-empty sets can have unique lowest members {well-ordered set}.

value

Sets can have unique lowest {least element} member. Sets can have unique highest {greatest element} member. Maximum at lowest member {least upper bound, element} and minimum at highest member {greatest lower bound, element} bound interval.

partition of set

Sets can split {partition, set}| into mutually exclusive subsets.

symmetric in sets

Relation between second coordinate and first coordinate can be image of relation between first coordinate and second coordinate {symmetric}. Relations can be reflexive and transitive {antisymmetric}, so both members are equal.