Outline of Number Theory
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MATH>Number Theory

number theory

Mathematical theories {number theory} can be about number relations and types.

continuum in number theory

Number of real numbers {continuum} is greater than number of natural numbers.

number types

Natural numbers are counting numbers: 0, 1, 2, 3, ... Counting numbers can be binary numbers. Integers are natural numbers plus their negatives: ... -3, -2, -1, 0, 1, 2, 3, ... Zero is its own negative. Rational numbers are integer fractions, such as -3/-1, -3/1, 3/-1, 3/1, -1/-3, -1/3, 1/-3, 1/3, and 0/integer, but not integer/0, because it has no definition. Rational numbers are repeated decimal numbers. Irrational numbers are non-repeating decimal numbers. Infinite operation series, making digit series, can represent irrational numbers {countable irrational number}. The number pi and all rational-number roots are countable irrational numbers {countably infinite}. However, most irrational numbers are not countable. Real numbers are rational numbers plus irrational numbers.

countable infinity

Numbers of natural numbers, integers, rational numbers, and countable irrational numbers are equal, because they can be put in one-to-one correspondence with counting numbers. See Figure 1.

Fraction has integer numerator and denominator. Countable irrational numbers are countable, because you can count operations and digits. Therefore, natural, rational, or countable irrational numbers have one-to-one correspondence with counting numbers.

continuum

Try to align natural numbers with non-countable irrational numbers in one-to-one correspondence by making an array, with natural numbers on one axis and non-countable irrational numbers on other axis. See Figure 2.

Mark diagonal, to take diagonal slash. See Figure 3.

Change the mark at first-row-and-column first position, at second-row-and-column second position, and so on. See Figure 4.

The new sequence is non-countable irrational number, because it randomly comes from the non-countable irrational-number array. However, it is not the same as any row or column sequence, because it differs from first row and column at first number, differs from second row and column at second number, and so on. If natural numbers and non-countable irrational numbers have one-to-one correspondence, the list must contain all possible non-countable irrational numbers. Therefore, natural numbers and non-countable irrational numbers have no one-to-one correspondence, and number of non-countable irrational numbers, and so number of real numbers, is greater than number of natural numbers.

Figure 1

Natural Numbers:	0	1	2	3	4	5	6	...
Integers:	0	-1	1	-2	2	-3	3	...
Rational numbers:	n0	n1	n2	n3	n4	n5	n6	...
Countable Irrational Numbers:	n0	n1	n2	n3	n4	n5	n6	...

Figure 2

Natural number	Irrational number expressed in binary coding
0	1 1 1 1 1 1 1 1 . . .
1	0 0 0 0 0 0 0 0 . . .
2	1 0 1 0 1 0 1 0 . . .
3	0 1 0 1 0 1 0 1 . . .
4	1 0 0 1 0 0 1 0 . . .
5	0 1 0 0 1 0 0 1 . . .
6	0 0 1 0 0 1 0 0 . . .
7	1 0 0 0 1 0 0 0 . . .
.	. . . .
.	. . . .
.	. . . .

Figure 3

Natural number	Irrational number
0	**1** 1 1 1 1 1 1 1 . . .
1	0 **0** 0 0 0 0 0 0 . . .
2	1 0 **1** 0 1 0 1 0 . . .
3	0 1 0 **1** 0 1 0 1 . . .
4	1 0 0 1 **0** 0 1 0 . . .
5	0 1 0 0 1 **0** 0 1 . . .
6	0 0 1 0 0 1 **0** 0 . . .
7	1 0 0 0 1 0 0 **0** . . .
.	. . . .
.	. . . .
.	. . . .

Figure 4

from Figure 3 diagonal
1 0 1 1 0 0 0 0 . . .

to new sequence by changing each position
0 1 0 0 1 1 1 1 . . .

magic square

Square number arrays {magic square}| can have row, column, and long-diagonal elements that add to same number.

nested interval

Smaller intervals {nested interval}| can be in larger intervals. For nested-interval series, interval length goes toward zero, and interval converges on a number {nest, interval} at a point {final residue}. Two numbers cannot be in same nest, because length greater than zero always separates them.

number line

Lines {number line} can have a zero point, positive numbers on right side in increasing order, and negative numbers on left in decreasing order.

parity of numbers

Two numbers can both be even or both be odd {same parity} or one be even and one odd {opposite parity} {parity, number}|.

ratio

Numbers with same units can divide {ratio}. Ratios have no units. Multiplying ratios makes ratio. $a/b > c/d$ if $a^M > b^N$ and $d^N > c^M$, where M and N are positive integers. $a/b < c/d$ if $a^M < b^N$ and $d^N < c^M$. $a/b = c/d$ if a/b !> c/d and a/b !< c/d. Irrational numbers are rational-number limiting values, as M and N become large.

MATH>Number Theory>Factoring

factoring numbers

Smaller numbers, excluding one, can divide into whole numbers with no remainders {factoring, numbers} {factor, number} {associate, number}. Most whole numbers {composite number} have factors.

Theories {theory of ideal numbers} {ideal numbers theory} can find unique factorization into primes.

divisible by 11

If integer is divisible by 11, sum of digits with alternating sign is 0. For example, 121 (1 - 2 + 1 = 0) is divisible by 11, but 1234 (1 - 2 + 3 - 4 = -2) is not divisible by 11.

divisible by 3

If integer is divisible by 3, sum of digits is divisible by 3. For example, 24 (2 + 4 = 6) is divisible by 3, but 38 (3 + 8 = 11) is not divisible by 3.

divisible by 5

If integer is divisible by 5, last digit must be 0 or 5. For example, 25 and 30 are divisible by 5.

fundamental theorem of arithmetic

All natural numbers, except the number one, factor into primes in only one way {fundamental theorem of arithmetic}.

greatest common factor

Two numbers have largest factor in common {greatest common factor, number}. First divide smaller number into larger. Then divide remainder into smaller. Then divide new remainder into first remainder. Continue until remainder is zero. Greatest common factor is remainder obtained just before remainder is zero. Greatest common factor is product of shared prime factors.

MATH>Number Theory>Field

closure of numbers

Adding or multiplying two real numbers makes real number {closure}.

identity element of number

Elements {identity element, number}, such as 0 or 1, can add to numbers to give same number or multiply number to give same number.

inverse element of number

Real-number reciprocals {inverse element, number} are real numbers.

quotient field

Rational numbers form mathematical field {quotient field} with addition, subtraction, multiplication, and division.

MATH>Number Theory>Form

form for number polynomial

Numbers can be polynomial expressions {form, number polynomial} with integer constants and variable coefficients. Different equivalent forms can represent same number. Forms can multiply {composition, form}.

geometric theory of number

Lattices and number forms can solve number-theory problems {geometric theory of numbers}.

rectangular form

Complex-number forms {rectangular form} can be x + i*y.

MATH>Number Theory>Magnitude

magnitude of number

Real or complex numbers have absolute values {magnitude, number}.

absolute value

Removing negative sign can find real-number magnitude {absolute value, number}. Absolute-value symbol is vertical lines around value. For example, absolute value of -17 equals 17: |-17| = 17. For complex numbers, absolute value is |a + b*i| = ((a + b*i) * (a - b*i))^0.5 = (a^2 + b^2)^0.5.

numerical value

Number values {numerical value} can be absolute values.

MATH>Number Theory>Notation

notation system

Real numbers can use notation {system of notation} {notation system}. For example, at positions, number-system base has powers. For example, 1234 = 1 * 10^3 + 2 * 10^2 + 3 * 10^1 + 4 * 10^0.

positional notation

Positions around decimal point can correspond to powers of number-system base {positional notation}|. For example, 1234 = 1 * 10^3 + 2 * 10^2 + 3 * 10^1 + 4 * 10^0.

MATH>Number Theory>Notation>Scientific Notation

scientific notation

Large or small numbers can use notation {scientific notation}| with mantissa between 1 and 10 times number 10 to characteristic power. For example, 120,000,000 = 1.2 x 10^8.

characteristic number

Scientific notation uses mantissa between 1 and 10 times number 10 to power {characteristic, number}.

mantissa

Scientific notation uses numbers between 1 and 10 {mantissa} times 10 to characteristic powers. Mantissa has number of significant digits.

MATH>Number Theory>Number System

binary number system

Number systems {binary number system}| {bicimal system} can use two as base. Base-2 number system uses two digits, 0 and 1. For example, 111 in base 2 is $1 * 2^2 + 1 * 2^1 + 1 * 2^0$, which equals 7 in base 10.

decimal number system

Base-10 number system {decimal number system}| {denary number system} uses ten digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9. For example, 1357 equals $1 * 10^3 + 3 * 10^2 + 5 * 10^1 + 7 * 10^0$.

duodenary number system

Base-12 number system {duodenary number system} uses twelve digits: 0, 1, 2, 3, 4, 5, 6, 7, 8, 9, a, and b.

modulo n

Number systems {modulo n} {mod n} can include only positive integers from number zero up to number {modulus, number} minus one. For example, modulo-3 number system has only integers 0, 1, and 2. For mod n, modulus is n.

sexagesimal as sixty

Things can have sixty parts {sexagesimal}.

unary notation

Amounts or values can be string lengths {unary notation}. Unary notation uses only one symbol, such as 1. For example, 1111 in base 1 has length 4, which equals 4 in base 10.

MATH>Number Theory>Theorem

analytic number theory

Analysis can assist number theory {analytic number theory}.

Dedekind cut

Rational numbers define cuts {Dedekind cut} in rational-number ordered set, so left side is less than or equal to rational number and right side is greater than rational number. Real numbers are limits of Dedekind-cut convergent sequences. Limits are rational-approximation converging-sequence limits. Irrational numbers partition rational-number sets.

Euler conjecture

For integer $n > 2$, $x1^{(n + 1)} + x2^{(n + 1)} + ... + xn^{(n + 1)} = z^{(n + 1)}$, where z and x1 through xn are integers, has no positive integer solutions {Euler conjecture}. Case $x1^4 + x2^4 + x3^4 = z^4$ $(n + 1 = 4)$ is false. Case $x1^5 + x2^5 + x3^5 + x4^5 = z^5$ $(n + 1 = 5)$ is false. Fermat's last theorem is case $x1^3 + x2^3 = z^3$, where $n = 2$ $(n + 1 = 3)$, which is true.

Fermat last theorem

For integer $w > 2$, $x^w + y^w = z^w$, where x y z are integers, has no positive integer solutions {Fermat's last theorem} {Fermat last theorem}. Fermat proved that $x^3 + y^3 = z^3$ has no positive integer solutions. Andrew Wiles proved theorem for all cases [1993].

properties

$x^2 + y^2 = z^2$ has solutions for x = 3, 4, 5, and so on. z is odd, x is odd, and y is even. $x + y > z$. In parameterization, $x = 2*m*n$, $y = m^2 - n^2$, and $z = m^2 + n^2$.

Multiples of Pythagorean triples are Pythagorean triples. For lowest Pythagorean triples, $z - y = 1$ if x is odd, or $z - y = 2$ if x is even. For lowest Pythagorean triples, $n = 1$, if lowest of x or y is even, or $m - n = 1$, if lowest of x or y is odd.

$x^2 + y^2 = z^2$ has $(x^2)/(z^2) + (y^2)/(z^2) = 1$ and $((x^2 + y^2)^{0.5})/z = 1$, so one percent plus other percent = 100%. $x^2/z^2 + y^2/z^2 + 2*x*y/z^2 = 1 + 2*x*y/z^2$ means $(x + y)^2 = z^2 + 2*x*y$, where $(x + y)^2$ is area of square whose side is straight line of $x + y$, and $2*x*y$ is two times area of triangle rectangle.

triangle

For $x + y = z$, three natural numbers lie on a straight line, with xy angle 180 degrees. For $x^2 + y^2 = z^2$, three natural numbers lie on a right triangle, with xy angle 90 degrees. Perhaps, for $x^3 + y^3 = z^3$, three natural numbers lie on a triangle with xy angle 60 degrees, but this only allows $x = y = z$, so no natural number solutions. Perhaps, for x^4 + y^4 = z^4, three natural numbers lie on a triangle with xy angle 45 degrees, but this only allows x < x and z > y, so no natural number solutions.

cube

x^3 + y^3 = z^3 makes (x + y)^3 = z^3 + 3*x^2*y + 3*x*y^2. 3 * x^2 * y + 3 * x * y^2 = 3 * x * y * (x + y). (x + y)^3 is volume of cube with side x + y. 3 * x * y * (x + y) is three times volume of rectilinear solid with sides x, y, and x + y. x + y > z. Perhaps, side lengths and angles make an impossible figure. Perhaps, all higher powers make impossible figures. Perhaps, x^3 + y^3 = z^3 requires not odd and even properties, but three-part system with divisible-by-3 numbers, numbers one higher, and numbers two higher. x, y, and z must come from different categories. Perhaps, this is impossible.

Pell equation

x^2 - A * y^2 = 1 {Pell's equation} {Pell equation}, where A is integer.

permanence of form

Rules for operations on integers can be for all algebras {permanence of form}.

uniqueness law

One and only one number adds or multiplies number to give another number {uniqueness law, number}.

Waring theorem

Integers are sums of at most nine cubes {Waring's theorem} {Waring theorem}.

MATH>Number Theory>Theorem>Axiom

Archimedes axiom number

If number is greater than zero, smaller number added to itself enough times can equal the number {axiom of Archimedes, number} {Archimedes axiom, number}.

calculation axiom

Number-theory axioms {calculation axiom} can be about calculation, such as associative law, commutative law, and distributive law.

completeness axiom

Adding anything to real numbers makes all preceding axioms untrue, so real-number system cannot be larger {axiom of completeness} {completeness axiom}.

connection axiom

Number-theory axioms {connection axiom} can be about operations, such as closure, uniqueness, and identity.

continuity axiom

Number-theory axioms {continuity axiom} can be about continuity, such as axiom of Archimedes and axiom of completeness.

order axiom

Number-theory axioms {order axiom} can be about order, such as transitive law. For two different numbers, one number is greater and one number is smaller. If first number is greater than second number, then first number plus third number is greater than second number plus third number. If first number is greater than second number, then first number times third number is greater than second number times third number.

MATH>Number Theory>Theorem>Factorial

Stirling theorem

If n > 7, n! ~ n^n * e^-n * (2 * pi * n)^0.5, where n is integer, and e is base of natural logarithms {Stirling's theorem} {Stirling theorem} {Stirling's formula}.

Wilson theorem

Numbers equal to (p - 1)! + 1 are divisible by p if and only if p is prime {Wilson's theorem} {Wilson theorem}.

MATH>Number Theory>Theorem>Order Relation

order relation
Rational numbers have order {order relation}.

consistency relation
If a is less than b, then a + c is less than b + c for all c, and a*c is less than b*c for all positive c {consistency relation}, where a b c are rational numbers.

density relation
If a is less than b, some c is greater than a and less than b {density relation}, where a b c are rational numbers.

extension relation
For interval from b to c, some a are less than c and greater than b {extension relation}, where a b c are rational numbers.

transitivity relation
If a is less than b, and if b is less than c, then a is less than c {transitivity relation} {transitive law} {transitivity, number}, where a b c are rational numbers.

trichotomy relation
For rational numbers a and b, a is greater than b, equal to b, or less than b {trichotomy relation}.

MATH>Number Theory>Theorem>Prime Number

prime number theorem
Ratio between number of prime numbers less than or equal to an integer and integer can have an approximation {prime number theorem}. Number of primes not exceeding number n is PI(n), whose limit can find the prime numbers: limit of PI(n) / (n / log(n)) = 1, as n goes to infinity.

Euclid theorem
Number of primes is infinite {Euclid's theorem} {Euclid theorem}.

Fermat theorem
If p is prime, and a is an integer with no common factor with p, then a^(p - 1) / p has remainder one {Fermat's theorem} {Fermat theorem}.

Goldbach hypothesis
Positive even integers are sums of two primes {Goldbach's hypothesis} {Goldbach hypothesis} {Goldbach's conjecture}.

Levy conjecture
Odd numbers can be primes plus two times primes {Levy's conjecture} {Levy conjecture}: p' + 2 * q' = 2*n + 1, where n goes from 0 to infinity.

Shor algorithm
Algorithms {Shor's algorithm} {Shor algorithm} can find prime factors.
modular
Modular arithmetics have circular sets of numbers. Mathematical operations are periodic.
process
Start with mod. Using any number smaller than the mod, take its first, second, and so on, powers and express result in the mod until number sequence shows a repeating pattern. Distance between repeats is period. Divide period by two and use result as mod exponent. If period divided by two is not even number, start over.
factors

Take the integers one above and one below result. Find largest common divisor of number and two integers to calculate number factors.

sieve of Eratosthenes

From natural-number list, cross out all second numbers except for number two, then cross out all third numbers except for number three, and so on {sieve of Eratosthenes} {Eratosthenes sieve}. What remains are prime numbers.

MATH>Number Theory>Number Types

algebraic number

Polynomial equations with rational coefficients have numbers {algebraic number} as roots, because polynomial functions involve only arithmetic operations. Algebraic numbers can have degrees, such as square root, depending on equation degree.

Kummer

If a is imaginary pth root of unity {algebraic number, Kummer}, f(a) = A(0) + A(1) * a + A(2) * a^2 + ... + A(p - 2) * a^(p - 2).

infinity

Algebraic and rational numbers have same infinity order.

denumerability

Integer-coefficient polynomial-equation roots are denumerable. Real-number-coefficient polynomial-equation roots are not denumerable. Algebraic numbers do not have unique factorization.

amicable number

For number pairs {amicable number}, one number's factor sum, including 1, can equal other number. For example, 6 is amicable with 6: 3 + 2 + 1 = 6. 9 is amicable with 7: 3 + 3 + 1 = 7.

Bernoulli number

Number series {Bernoulli number} can be 1/6, 1/30, 1/42, 1/30, 5/66, 691/2730, 7/6, 3617/510, ...

Catalan number

Number series {Catalan number} can be 1, 2, 5, 14, 42, 132, 429, 1430, 4862, ...

cubic number

Numbers {cubic number} that are cubes of n (n^3) equal sum of n consecutive odd numbers starting with n * (n - l) + l and equal n * (n * (n - 1) + l) + 2 * (n - 1).

Euler number series

Number series {Euler number series} (En) can be 1, 5, 61, 1385, 50521, ...

Euler number

e = 2.7182818284... {Euler's number} {Euler number} {Napier's constant}.

even number

Whole numbers {even number} can be multiples of two.

Fibonacci number

Numbers {Fibonacci number} {Fibonacci sequence} can be sums of the two preceding numbers, starting with one: 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89, ... Fibonacci sequence is a geometric progression in which number ratios converge on golden ratio: 2/1, 3/2, 5/3, 8/5, 13/8, and so on.

gamma as constant

Limit of 1/1 + 1/2 + 1/3 + ... + 1/n - log(n) is 0.5772... {Euler's constant} {gamma}, where n is integer that goes to infinity. Is Euler's constant rational or irrational?

gnomonic number

Odd numbers {gnomonic number}, 2*n + 1, can add to squares of natural numbers n^2 to make squares of next natural number (n + 1)^2: n^2 + 2*n + l = (n + 1)^2.

Godel number

Gödel numbering {Gödel number} can assign unique digit strings to statements, ideas, images, strings, and so on.

golden ratio

For Fibonacci sequence, sum of two numbers divided by larger number equals larger number divided by smaller number {golden ratio}| {golden section}. Golden ratio equals 1.618034..., reciprocal equals 0.618033..., and square equals 2.618034....

construction

First, bisect square side. Then draw circle with center at bisection point and radius from bisection point to square far corner. Extend one square side until extension meets circle. Extended side has length 1.618...

geometry

If right triangle has one side equal 1 and hypotenuse equal 1.618..., angle is one radian. Golden rectangle has sides in golden ratio and has central point angle of 58 degrees. Pentagram and decagram have sides in golden ratio. Golden ratio is ratio of rectangle {golden rectangle} sides in logarithmic spirals.

music

In music, ratio 2^0.67 = 1.59 ~ 1.618... is similar to major-sixth/octave = 1.67, octave/major-fourth = 1.6, and minor-seventh/major-second = 1.59. Golden ratio and its inverse can make all music harmonics.

ideal number

If integer sets form mathematical fields, product of any two integers a b, plus product of any other two integers c d, is an integer e {ideal number}: a * b + c * d = e. Ideal numbers have unique factorization, primes, and maxima. Ideal numbers are algebraic-number classes.

integer

To count numbers greater than zero {positive number} or less than zero {negative number} use signed whole numbers and zero {integer}, such as -9, 0, +9.

Integers can be function domain J, and positive integers can be function domain J+.

sums

Positive integers are sums of at most four squares. Integers are sums of at most 19 quarts. Positive even integers are sums of not more than four primes. Odd integers are sums of not more than three primes.

primes

Are there infinitely many positive integers, so both one less and one more are prime?

irrational number

Numbers {irrational number}| can have infinite non-repeating decimals. Example is 2^0.5. Polynomial positive roots can have irrational numbers. Most irrational numbers are transcendental numbers, such as pi and e, not algebraic numbers. Transcendental-number infinity order is more than algebraic-number or rational-number infinity order. Irrational numbers in closed intervals are rational-number-series limits.

natural number

To count how many, use whole numbers, such as one, two, and so on {natural number}| {cardinal number}. Zero is natural number.

normal number

Real numbers {normal number} can have all digits, pairs, triples, and so on, in equal numbers.

odd number

Whole numbers {odd number} can be not multiples of two.

ordinal number

To count in order, use numbers {ordinal number}| like first, second, third, and so on.

perfect cube

Numbers {perfect cube} can be natural-number cubes, such as 1 = 1^3, 8 = 2^3, and 27 = 3^3.

perfect number

Numbers {perfect number} can equal sum of prime factors: 6 = 1 + 2 + 3. If 2^n - 1 is prime number and n is integer, 2^(n - 1) * (2^n - 1) is perfect number. Number's prime-factor sum can be greater than number {deficient number} {defective number}. Number's prime-factor sum can be less than number {abundant number} {excessive number} {redundant number}.

perfect power

Numbers {perfect power, number} can be smaller-number nth powers.

perfect square

Numbers {perfect square} can be natural-number squares, such as 1 = 1^2, 4 = 2^2, and 9 = 3^2.

prime number

Only one and the number can divide into some whole numbers {prime number}| without remainder. For every prime p, factorial of p minus one, plus 1, is factorable by prime: ((p - 1)! + 1) / p.

quadratic irrational number

General quadratic equation (a*x^2 + b*x + c = 0) solutions {quadratic irrational number} can have form a + b^0.5, where a and b are rational and b is not a perfect square. Ruler and compass constructions that do not result in rational numbers can most simply result in quadratic irrational lengths. Quadratic irrational numbers can be periodic continued fractions. For a = 0 and b = 14, 14^0.5 = 3 + (1 + (2 + (1 + (6 + (1 + (2 + (1 + (6 + (...)^-1)^-1)^-1)^-1)^-1)^-1)^-1)^-1)^-1. Solutions have form A + (B + (C + (D + (E + (B + (C + (D + (E + (...)^-1)^-1)^-1)^-1)^-1)^-1)^-1)^-1)^-1.

rational number

Numbers {rational number}|, such as 3/5 or 1/9, can have finite or repeating decimals. In one system, rational numbers derive from natural numbers, using reflexive, symmetric, and transitive axioms. Rational numbers are all numbers expressed as one integer divided by another integer.

real number

Numbers {real number}| can include both irrational and rational numbers. Real numbers make all equation roots. In one system, real numbers derive from natural numbers using axioms of connection, calculation, order, and continuity. Real numbers and points on closed intervals have one-to-one correspondence. Real numbers and points in closed-interval squares have one-to-one correspondence, because square points are real-number pairs and line points are numbers that alternate pair digits.

square number

N^2 = 1 + 3 + 5 + ... + (2*N - 1) {square number, theory}. (N^2 + N) / 2 = sum from 1 to N of i = (N/2) * (N + 1).

transcendental number

Non-algebraic numbers {transcendental number}| are roots of equations with trigonometric, inverse trigonometric, exponential, and logarithmic functions. Transcendental numbers can relate to circles, triangles, exponents, and logarithms, such as pi = 3.1415926535... and e = 2.7182818284... = Euler's number.

pi

pi is the limit of ratio between many-sided regular-polygon circumference and center-to-vertex line length.

e

e is base of expression e^-x, such that derivative of e^-x equals e^-x. Lower values make lower derivatives and higher make higher, so e is in middle. It also makes x^(1/x) maximum. It is the limit of (1 + 1/n)^n, when compound interest has many periods and interest is 1/n per period. Definite integral of (1/x) * dx from 1 to e equals 1.

e = 1/0! + 1/1! + 1/2! + 1/3! + ... = 1/1! + 2/2! + 3/3! + 4/4! + ...

e^a = 1 + a + a^2/2! + a^3/3! + ...

-1/(e^pi) = 1 + i - 1/2 - i/6 + 1/24 + i/120 + ...

-1/(e^pi) - 1/2 + 1/24 - 1/720 + ... = i*(1 - 1/6 + 1/120 + ...).

i = (-1/(e^pi) - 1/2 + 1/24 - 1/720 + ...)/(1 - 1/6 + 1/120 + ...).

integral from -infinity to +infinity of e^-x^2 * dx = pi^0.5.

trigonometry

sin(a) = a - a^3/3! + a^5/5! - a^7/7! + ...
cos(a) = 1 - a^2/2! + a^4/4! - a^6/6! + ...

i

i = e^(i*pi/2).
ln(i) = i*pi/2
e^(i*a) = cos(a) + i*sin(a), where a is in radians and is real.
sin(a) = (e^i*a - e^-i*a)/2i
cos(a) = (e^i*a + e^-i*a)/2.
e^i = 1 + i - 1/2 - i/6 + 1/24 + i/120 + ...
cos(i) = 1 + 1/2 + 1/24 + 1/720 + ...
i = arccos(1 + 1/2 + 1/24 + 1/720 + ...).
-e^-pi = e^i
i = ln(-e^-pi).
e^-pi = -e^i
pi = -ln(-e^i) = - ln(-1) - ln(i).
ln(i) = i*pi/2 = i/2 * (- ln(-1) - ln(i)).
ln(i) = -i*ln(-1)/2 - i*ln(i)/2
i*ln(i) = ln(-1)/2 + ln(i)/2
2*i*ln(i) = ln(-1) + ln(i).
2*i*ln(i) - ln(i) = ln(-1).
ln(i) * (2*i - 1) = ln(-1).
2^0.5 is minimum of rectangle-diagonal-length to average-side-length ratio.

transfinite number

The Hebrew-alphabet first letter, aleph, with subscript from 0 to infinity, represents infinity orders {transfinite number}|. The infinity orders are infinite. Finite-number sets have a greatest number.

trillion

10^12 is a number {trillion}|.

MATH>Number Theory>Number Types>Complex Number

complex number

Numbers {complex number}| can have real part x added to imaginary part i*y, where y is real number: x + i*y. Complex numbers can solve all polynomial equations, such as x^2 + 1 = 0. Complex numbers are roots of homogeneous polynomial equations with only positive factors: a*x^n + ... + C = 0. For example, quantum mechanics has only positive energy components, and so uses complex-number equations. Because polynomials can approximate all equations, complex numbers can approximately solve all equations. Because they have two independent components, complex numbers have no inequality equations.

imaginary number

The number i equals -1^0.5 {imaginary number}|. By DeMoivre's theorem, any power of i is expressible as a + b*i. For example, i^2 = -1, i^3 = -i, and i^4 = 1. i^0.5 = 1/(2^0.5) + (1/(2^0.5))*i. i^0.5 = -1/(2^0.5) - (1/(2^0.5))*i. i^0.333 = (3^0.5)/2 + (1/2)*i. Therefore, complex numbers need only a real part and an imaginary part, with no other components.

i^i = e^-pi / 2, for log i = 0.5 * pi * i. All i^i are real numbers. z = r * e^i*A. log z = log r + i*A. e^i*A = cos A + i*sin A.

All polynomial roots are expressible by at least one complex number (though not as radicals, by the Abel-Ruffini theorem [1824]).

Perhaps, a new complex-number type can use a factor of reals and imaginaries, but not be a hypercomplex number.

Argand diagram

Complex numbers can be on planes {Argand diagram}, with real numbers on horizontal axis and imaginary numbers on vertical axis. Complex numbers can be on planes with polar coordinates: z = r * cos(A) + i * r * sin(A), where r equals length from point to origin {absolute value, complex number} {magnitude, complex number} {norm, complex number} {modulus, complex number}, and A equals angle to horizontal axis {argument, complex number} {phase, complex number} {amplitude, complex number}.

complex conjugate

Complex numbers x + i*y have associated complex numbers {complex conjugate}|: x - i*y. Complex numbers multiplied by complex conjugates make real numbers whose magnitude is complex-number squared.

DeMoivre theorem

(cos(A) + i * sin(A))^n = cos(n*A) + i * sin(n*A) {DeMoivre's theorem} {DeMoivre theorem}.

Euler identity

e^i * pi = -1 {Euler's identity} {Euler identity}.

MATH>Number Theory>Number Types>Complex Number>Hypercomplex

hypercomplex number

Numbers can have more than one imaginary component {hypercomplex number} {hypernumber}. Complex numbers are two-dimensional vectors, and hypernumbers are n-dimensional vectors. Hypernumbers can represent tensors, quaternions, matrices, determinants, and all number types. Hypernumbers are directed line segments {extension, calculus}.

magnitude

Magnitudes are the same as for vectors.

addition

Hypernumbers add corresponding parts, like complex numbers.

multiplication

Hypernumbers, like complex numbers, multiply like polynomials. Products are scalars or vectors. Product of same axis and itself makes scalars. When axis multiplies another axis, result is vector orthogonal to both original axes.

quaternion

Hypercomplex numbers {quaternion} can be scalar plus three-dimensional vector: a + b*i + c*j + d*k, where a, b, c, and d are real numbers, and i, j, and k are orthogonal unit vectors.

operations

Quaternion addition is like translation. Multiplying quaternions is non-commutative: i*j = k, j*k = i, k*i = j, j*i = -k, k*j = -i, i*k = -j and describes quaternion rotations. Quaternions can divide.

space

Complex numbers map to two-dimensional space, and quaternions map to three-dimensional space.

spinor

Real-number spinors represent rotating quaternions.

biquaternion

Hypernumbers {biquaternion} can be real quaternion plus w times real quaternion: a + b*i + c*j + d*k + w * (e + f*i + g*j + h*k), where w^2 = 1. w commutes with all real quaternions. Biquaternion operations obey multiplication product law and are linear, associative, and non-commutative.

octonion

Hypernumbers {octonion} can have one real term and seven imaginary terms: N, i, j, k, l, m, n, p. Imaginary term multiplied by itself gives real term. Two different imaginary terms multiply to different third term, by cyclic ordering: i * j = k, for example. Octonions can divide. Figures {Fano plane} that represent octonions have seven points, each with two links.

MATH>Number Theory>Number Types>Complex Number>Law

parallelogram law

Adding complex numbers is like adding vectors {parallelogram law}. Adding is translation. Triangle 0, 1, w is similar to triangle 0, z, wz.

similar triangles law

Multiplying complex numbers is like multiplying vectors {similar triangles law}. Multiplying two complex numbers multiplies moduli and adds arguments. Arguments are like logarithms in this way.

MATH>Number Theory>Number Types>Factorial

factorial number

N! = N * (N - 1) * ... * 1 {factorial}|. Factorial symbol {factorial sign} is exclamation point.

interpolation problem

Factorial is true even if number n is not integer: n! = product from k = 1 to k = infinity of ((k + 1) / k)^n * k / (k + n) {interpolation problem}. Eulerian integrals, like gamma function or beta function, can be for interpolation.

MATH>Number Theory>Number Types>Pi

Leibniz formula

pi = 4 * (1/1 - 1/3 + 1/5 - 1/7 + ...) {Leibniz formula}.

Wallis product

pi = 2 * (2/1) * (2/3) * (4/3) * (4/5) * (6/5) * (6/7) * ... pi/2 is product {Wallis's product} {Wallis' product} {Wallis product} of terms (2 * r / (2*r - 1)) * (2 * r / (2*r + 1)), from r = 1 to infinity.

MATH>Number Theory>Numeral

Arabic numeral

Number representations {Hindu-Arabic numeral} {Arabic numeral}| can use different systems, such as derivatives of Hindu numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.

digit as numeral

Number representations {digit, numeral} {numeral, digit} can use different systems, such as Arabic numerals: 0, 1, 2, 3, 4, 5, 6, 7, 8, and 9.