Outline of Logic
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MATH>Logic

logic mathematics

Consistent and complete valid inference rules can cover all reasoning situations {logic}. Logic laws are language laws, not necessarily laws of reality.

Traditional logic started with statement and performed conversion to get converse, obversion to get obverse, contraposition to get contrapositive, and inversion to get inverse {immediate inference, tradition}. Logic is about qualities and relations, not about world.

People learn logical principles, as people perceive particular examples. Babies and young children do not know or use logical principles.

Logic and mathematics provide proposition forms but not actual propositions.

scope

Statements have terms and operations {scope, logic}.

randomness

It is impossible to prove that number is random. By information theory, numbers have same information content as all other numbers. High randomness means high information complexity.

category mistake

Explanation errors {category mistake}| can compare different-type things, compare different levels, or create wrong-type or at wrong-level categories.

consistency in logic

Statement sets, such as arguments, can be true {consistency, logic} if and only if all statements are true. Sound arguments are consistent.

In consistent formal systems, no proposition can be both true and false, and propositions are either true or false. Logical consistency requires model or universe in which all sentences are true {consistency theorem}.

omega

Propositions can depend on variables. Propositions can be true for only some variable values {omega consistency, logic} and not true for others. Incompleteness theorem states that the proposition that formal system is omega consistent is not provable by the formal system.

contradiction

If statements are not consistent, at least one statement is false {contradiction}.

cut elimination theorem

Sequent-calculus proofs can be truth-trees or truth-tables, which always eliminate formulas {cut elimination theorem}.

decidability

Proofs show that either proposition or its negation is true {decidability}|.

defeasable

Proofs can show that statement is false or needs revision {defeasable}.

equivalence relation

Relations {equivalence relation, symmetry} can be symmetric, reflexive, and transitive. Two propositions can imply each other {material equivalence}. Material equivalences {logical equivalence, logic} can be tautologies.

predicative theory

Abstract objects exist only if they have predicative definitions {predicative theory}. Predicative definition must be countable.

propositional attitude

Mind and mental states use thoughts, perceptions, emotions, and moods {propositional attitude, logic}, which associate phenomenon with representation or intentional content. Judgment is proposition and is separate from truth, falsity, or other claim or feeling about statement. Communication expresses belief, goal, intention, or emotion to audience that is to understand message and recognize speaker intention.

propositional sign

Complex-sentence words and clauses {propositional sign} {propositio} can indicate propositions.

recursion in logic

Processes can cycle or loop {recursion, logic}. Loop shows infinite process in finite way, using self-reference. Gödel's theorem uses recursion. Recursion can happen if something is both program and data.

reducibility axiom

For any function, equivalent type-0 propositional functions exist {axiom of reducibility, logic} {reducibility axiom, propositional function}. Equivalent type-0 propositional functions {relation, logic} have object sets {class, logic} as members. For example, functions can have two variables, whose pairs are class members.

reflection principle

People can assign meaning or interpretation to axiomatic systems {reflection principle}, to judge if axioms and rules are valid. This method can find mathematical truth beyond proofs derived in axiomatic system itself.

reflexivity
Things can relate to self {reflexivity}.

truth-function
Propositions can have functions {truth-function}. Proposition truth varies with function and arguments. Truth-functions have truth-values TRUE or FALSE. Truth-values can be in truth-tables.

truth-table
For connectives, all true or false term combinations can be in tables {truth-table}| whose cells indicate statement truth or falsity.

validity in logic
At argument inferences, applying correct rule makes conclusion true if premises are true {validity, argument}.

MATH>Logic>Applications

logic applications
Logic has applications {logic applications}. People can describe how and why they accepted proposition. Sets and logical operations have equivalences. Union is equivalent to AND. Intersection is equivalent to OR. Universal set is equivalent to TRUE. Empty set is equivalent to FALSE. Complementary set is equivalent to NEGATION or NOT.

algebra of propositions
Logical Boolean algebra can determine proposition validity {proposition algebra} {algebra of propositions}.

Boolean algebra
Logic algebras {Boolean algebra}| {extensional logic} can depend on set theory {logic of classes}.
operations
Rules for operating on sets are the same as logic rules. Boolean algebra uses the number zero as empty set for false and the number one as universal set for true. Boolean addition is set-theory selection operation. Boolean-algebra negation is set complement: -a = ~a = NOT a. Boolean-algebra addition is set union {inclusive OR, logic}: a + b = (a | b) = a OR b. Boolean-algebra subtraction is set intersection: a - b = (a & b) = a AND b.
laws
Boolean algebra follows set-theory contradiction, commutation, association, and distribution laws. Boolean algebra uses excluded-middle law.

number calculus
Number calculus is equivalent to logic calculus, if 0 equals truth-value false and 1 equals truth-value true {number, calculus}.

deontic logic
Statements using must and may have logic {deontic logic}.

logical calculus
Valid argument schemas {logical calculus} can use rules or syntax to move from simple valid arguments to complex ones, such as natural-deduction calculus (Gentzen) [1934] or tableau or truth-tree calculus (Beth) [1955]. For first-order logic, semantic proof is also syntactic proof {soundness theorem, logical calculus}.

predicate calculus
Predicates can have calculus {predicate calculus, logic} {calculus of relations}.
laws
Predicate calculus uses contradiction law, excluded-middle law, detachment rule, tautology, addition, association, permutation, and summation. AND, NOT, OR, ALL, SOME, and EQUIVALENT have meaning.
variables

Predicates can have variables. Predicate can have more than one variable {n-place predicate}.

first-order

Variable can be terms {first-order predicate}. For first-order predicates, constants are proper nouns, and variables are pronouns or common nouns {term, predicate}.

First-order predicate calculus is complete.

first-order: quantifiers

First-order predicate calculus {functional calculus} {first-order logic} {restricted predicate calculus} can have quantifiers. Quantifiers can affect variables {bound variable} or not {free variable}.

If A implies B, if A and B have bound variables, and if B, then every A value has a B value {generalization rule} {rule of generalization}. If A implies B, if A and B have bound variables, and if B implies A, then an A value exists {specification rule} {rule of specification}.

second-order

Variables can be predicates {second-order predicate}.

second-order: recursion

Predicates can contain themselves {recursive predicate}. Recursive predicates can assume existence of a set that does not actually exist and so have contradiction.

proof in logic

True formula {string, logic} sequences {canonical proof} {proof, logic} {demonstration, logic} go from premises to conclusions without errors. Canonical proofs establish mathematical-statement meaning. Non-canonical proofs establish canonical-proof possibility.

propositional calculus

Formal logic {propositional calculus, logic} {sentential calculus} can be about statements {proposition} that have one subject, one predicate, constants, and variables. Statements are propositional functions, with variable for subject and subject property for predicate. Assertion that proposition is true is a different statement than the proposition itself.

connectives

Propositional calculus uses NOT, OR, AND, IMPLIES or IF/THEN, and IF AND ONLY IF connectives. AND, NOT, and OR are constant operators.

quantifiers

Subject can have universal quantifier: for any x, subject has the property. Subject can have existential quantifier: at least one x has the property.

instantiation

Variables have possible-value sets {class, propositional calculus}. Values can substitute for subject variables {universal instantiation}. If arbitrarily selected values have a predicate property, class has property {universal generalization}. If class has property, value has property {existential instantiation}. If value has property, at least one thing in class has property {existential generalization}.

Scholastic method

Put all arguments into chain of syllogisms {Scholastic method}| to prove or refute answers.

semantic proof

Valid argument schemas can use semantics and try to find counterexamples. If argument finds none, it is proof {semantic proof}. For first-order logic, semantic validity is also syntactic validity {soundness theorem, semantic proof}. For first-order logic, if one can find semantic counterexample, syntactic calculus cannot prove argument. Second-order, monadic, modal, and temporal logics use semantic argument proofs.

sequent calculus

Natural deduction has calculus {sequent calculus}. Statement sequence gives reasoning chain and conclusion {sequent, reasoning}. Stating simple statements {basic sequent} in natural deductions starts premise or conclusion.

rules

Rules {introduction rule} can allow operation to make more complex formulas from simpler ones. Rules {elimination rule} can allow inference from complex formulas to simpler formulas. The reductio-ad-absurdum rule eliminates hypotheses. Sequent-calculus proofs can be truth-trees or truth-tables, which always eliminate formulas {cut elimination theorem, sequent}.

type theory

Sets and function-of-sets sets have object types {theory of types, logic} {type theory}.

purpose

Distinguishing between types avoids set-theory paradoxes.

types

Sets about objects have type 0. Sets about functions of type-0 sets have type 1. Sets about type-1 sets have type 2. Type-n sets are sets about type n-1 sets.

reducibility

For any type, an equivalent type-0 propositional function exists {axiom of reducibility, type theory} {reducibility axiom, type theory}. Equivalent type-0 propositional functions {relation, type} have classes as members. Classes have object sets. For example, functions can have two variables, and its class can have variable pairs as members.

class

Classes are similar if they have one-to-one correspondence. They are then reflexive, symmetric, and transitive.

MATH>Logic>Conditional

conditional statement

Statements can connect by IF/THEN, as in IF a THEN b {conditional statement, logic}. The only false conditional is when first statement is true and second is false. If antecedent is false, all consequents are true.

hypothetical

Conditionals are hypotheticals. If their statements do not relate, reasoning seems suspect. Such conditionals are typically not possible in universe, and conditional probability is typically zero.

not proposition

Perhaps, conditionals are not propositions and so are not true or false (V. H. Dudman).

apodosis

Antecedent precedes second statement {consequent, logic} {apodosis}.

protasis in logic

First statement {antecedent, logic} {protasis, logic} precedes consequent.

condition in logic

Conditions {condition} are necessary or sufficient. If X is not true, Y is not true {necessary condition}. If X is true, Y is true {sufficient condition}.

MATH>Logic>Conjunction

conjunction

Statements can connect by AND, as in a AND b {conjunction, logic}. If statement is true and second statement is true, statement "first statement AND second statement" is true: p and q, so (p & q). The only true conjunction is when both statements are true {conjunction rule} {rule of conjunction}. Statements in conjunctions can relate or be independent.

agglomeration

Conjunction can apply to functions of cases: f(A) AND f(B). Conjunction can apply to function cases: f(A AND B). If cases do not exclude each other, the two conjunctions are equal {agglomeration}: f(A) AND f(B) = f(A AND B).

MATH>Logic>Definition

definition in logic

Defining {definition} can state class and distinction.

impredicative definition

Definitions {impredicative definition}| {vicious-circle principle} can define objects in terms of object classes, a type of circular definition. The idea of set of all sets leads to such contradiction. Logical paradoxes can depend on impredicative definition.

operational definition
Definitions {operational definition, logic} can be how to use words.

predicative definition
Definitions {predicative definition} can not quantify over all class objects.

MATH>Logic>Fallacy

fallacy in logic
Reasoning can be incorrect, irrelevant, or ambiguous {fallacy}.

MATH>Logic>Fallacy>Ambiguity

accent fallacy
People can change word emphasis, inflection, or context {accent, fallacy}, an ambiguity fallacy.

amphiboly
Two grammatical word or phrase forms can be in different word groups, link to different pronouns, or be different speech parts {amphiboly}, an ambiguity fallacy.

composition fallacy
Extension to whole from part, or to class from individual, can have no logical basis {composition fallacy}|, an ambiguity fallacy.

decomposition fallacy
Extension to part from whole can have no basis {decomposition}, an ambiguity fallacy.

division fallacy
Idea about class can apply to individual, or idea about whole can apply to part {division fallacy}, an ambiguity fallacy.

either-or fallacy
Assuming only mentioned alternatives, if other alternatives exist, is incompleteness {either-or fallacy}|, an ambiguity fallacy.

equivocation fallacy logic
Words can have two meanings or have different contexts {equivocation}, an ambiguity fallacy.

false dichotomy
Assuming only two conclusions or premises, if many exist, is incompleteness {false dichotomy}| {either/or fallacy}, an ambiguity fallacy.

haec ergo quid fallacy
Thingness can differentiate thing from other things {haec ergo quid fallacy}. However, two things can share more similarities than differences. Thing natures can be general categories.

ignoring the base rate
Conditions have probabilities, such as 1% for diseases. Tests have reliability, such as 90%, and corresponding false-positive rates, such as 10%. If test is positive, most people think that chance of having condition is reliability, such as 90%, not probability divided by reliability, such as 0.9% {ignoring the base rate}.

misplaced concreteness
Wholes can be only one aspect {misplaced concreteness fallacy}|, an ambiguity fallacy. Part relations can be more important than part types.

scope fallacy

Words often have ambiguous scope or change scope after sentence rearranging or inference making {scope fallacy, logic}, an ambiguity fallacy. Statement, subject, or predicate negation changes scope. Reference change changes scope.

vagueness fallacy

Arguments can use imprecise language {vagueness}, an ambiguity fallacy.

MATH>Logic>Fallacy>Deduction

affirming the consequent

If A then B is true, and B is true, then A is true {affirming the consequent}|, a deduction fallacy.

begging the question

Arguments can assume conclusion in question without proof {begging the question}| {petitio principii}. It is a deduction fallacy and is the same as circularity. However, allowing conclusion with which no one argues is not fallacy. Proposing conclusions for argument's sake is not fallacious.

circular reasoning

Conclusion can prove premise {circular argument} {circular reasoning}| {circularity}. It is a deduction fallacy and is the same as begging the question.

confectionary fallacy

Granting permission for two things can sound like permitting first or second, like exclusive OR, but is actually conjunction {confectionary fallacy}, a deduction fallacy.

denying the antecedent

If A then B, and not A, so not B is incorrect logic {denying the antecedent}|, a deduction fallacy. If A then B, and not B, so not A is modus-tollens denying the consequent and is correct reasoning.

false analogy

People can incorrectly assume that if two things are alike in some ways, they are alike in other ways {false analogy}|, a deduction fallacy.

four-term fallacy

Syllogism can incorrectly have four terms, typically because one term has two meanings {four-term fallacy}, a deduction fallacy.

guilt by association

A is B; C is B; so A is C is association, not deduction {guilt by association}|, a deduction fallacy.

masked man fallacy

I know who my father is. I do not know who the masked man is. My father is not the masked man {masked man fallacy}|. However, the masked man can be anybody. Sentence objects are not the same. It is a deduction fallacy.

non sequitur

People can make wrong implications {non sequitur}|, a deduction fallacy.

omission fallacy

People can leave out important part {omission fallacy}, a deduction fallacy.

oversimplification

Overgeneralization, jumping to conclusion, stereotyping, either-or fallacy, or trivial analogies simplify argument {oversimplification}|, a deduction fallacy.

post hoc prompter hoc

Because effect follows something, the something can be said to be cause {post hoc ergo prompter hoc} {post hoc prompter hoc}, a deduction fallacy.

undistributed middle

In syllogism, neither premise can have distributed term, which applies to All or No {undistributed middle fallacy}|, a deduction fallacy.

vicious circle fallacy

Premise or definition can include the term to define {vicious circle fallacy}|, a deduction fallacy.

MATH>Logic>Fallacy>Irrelevance

accident fallacy

Event can be happenstance, rather than relevant {accident, fallacy}, an irrelevance fallacy.

ad hominem argument

Attacks can be on people instead of arguments, to discredit experts or people with bad reputations {ad hominem argument}| {argumentum ad hominem}, an irrelevance fallacy.

bandwagon effect logic

Appeals to follow popular opinion are not about argument {bandwagon effect}| {bandwagon appeal}, an irrelevance fallacy.

bias charge

Arguers can talk about people who believe statements, rather than reasoning for or against statements {bias charge}|. It is an irrelevance fallacy and is the same as ad hominem.

character assassination fallacy

Attacking personal character is not about the argument {character assassination fallacy}|. It is an irrelevance fallacy and is the same as ad hominem.

definist fallacy

Arguments can define terms in favorable or unfavorable ways without discussion {definist fallacy}, an irrelevance fallacy.

extension fallacy

People can exaggerate claim or include too much {extension fallacy}, an irrelevance fallacy.

false testimony

People can pay for, or coerce, testimony {false testimony}, an irrelevance fallacy.

ignoratio elenchi

Arguments can be about statement that is not statement to prove but only related statement {ignoratio elenchi}, an irrelevance fallacy.

ignoring the context

People can use omission or misinterpretation {ignoring the context}, an irrelevance fallacy.

ignoring the question

Leaving out part is not about the argument {ignoring the question}|. It is an irrelevance fallacy and is the same as red herring or name-calling.

irrelevance fallacy

Arguments can refer to something other than current premises and conclusions {irrelevance}, an irrelevance fallacy. Appeals to force, person, pity, the people, or authority are irrelevant fallacies. Arguments from ignorance, neglect of circumstances, questions containing implied question, and irrelevant conclusions are irrelevant fallacies.

loaded word

People can use words that cause emotional reactions {loaded word}, an irrelevance fallacy.

many-questions fallacy

Arguments can use valid inferences from answer to question that was about something non-existent and so not real {many-questions fallacy}, an irrelevance fallacy.

name-calling

People can apply false label or use emotional words or connotations without evidence {name-calling}. It is an irrelevance fallacy and is the same as red herring or ignoring the question.

overgeneralization

People can exaggerate claims or include too much {overgeneralization}| {hasty generalization}, an irrelevance fallacy. For example, Some A is B, C is A, so C is B.

red herring

People can use false issue, emotional issue, or digression {red herring}|. It is an irrelevance fallacy and is the same as ignoring the question or name-calling.

straw-man fallacy

Arguments can distort or exaggerate opponent ideas, making them easier to attack {straw-man fallacy}|, an irrelevance fallacy.

transfer of association

Arguments can use association with something that causes emotion {association transfer} {transfer of association}, an irrelevance fallacy.

MATH>Logic>Foundations

logic foundations

Approaches to logic are logicist, intuitionist, and formalist {logic foundations}.

formalism in logic

Universally accepted logical principles plus simple formal systems can establish logic and formal-system consistency {formalism, logic}. Formalism tries to establish arithmetic, number theory, and logic consistency and foundations, without set theory.

formula

Formalism uses symbolic expressions {formula} for logical relations. Formulas connect symbols using logic rules {well-formed formula} {wff} and therefore have syntax. Formulas have truth-values. Formulas have meaning though they have no words.

schema

Sentence or formula can use term or clause placeholders {schema, logic}. Schema is not true or false, until terms or clauses substitute for placeholders.

intuitionism in logic

Logic comes from mathematics {intuitionism}. Whole numbers come from time intuition. The only proof of existence is to make something exist. Truth is about provability or assertibility. In intuitionism, all definitions and proofs are constructive. Excluded-middle law can only work for proofs with finite numbers of steps. Double negation is not equivalent to original statement. Kripke trees can formalize intuitionist logic.

intuitionist logic

Statements can be true for observer, be false for observer, have later decision, or never have decision {intuitionist logic} {topos theory}. Observer actions can access different information and can affect truth. Shared observations have same truth. The same information always gives same truth-value.

logicism

Logic can be an axiomatic system {logicism}. Undefined terms are elementary proposition, propositional function, elementary-proposition truth assertion, proposition negation, and proposition disjunction {inclusive OR, logicism}. Syllogism rules are theorems.

MATH>Logic>Inference

inference in logic

Starting with statement or statements, argument {inference, logic} can derive further statements.

abductive inference

If an object belongs to a class and has probability of having a property, other class objects have probability of having the property {ampliative inference} {abductive inference} {abduction, logic}. Ampliative inference goes from one or more examples to abstraction {hypothesis, ampliative inference} that explains evidence. From observations and theoretical assumptions, abduction infers best explanation.

enthymeme

Inference can rely upon suppressed premise {enthymeme} {enthymematic}. Shortened categorical syllogism has two premises but no conclusion, because conclusion is obvious.

immediate inference

From one premise, inferences {immediate inference, logic} can be "All a are b" implies "No a are no b", "All a are b" entails "Some a are b", and "No a are b" entails "Some a are not b".

MATH>Logic>Necessity

logical necessity

Logical forms can appear to be true by necessity {logical necessity}, based on form alone. To test sentence truth, transform into logical forms.

nomic necessity

Necessity {nomic necessity} can be regular and law-like. Logically possible worlds can have same logic rules as universe.

MATH>Logic>Paradox

paradox

Seeming logic can lead to absurd or meaningless statements {paradox}. Paradox is about conflict of opposites, conflict with accepted ideas, or category conflict.

resolution

Paradoxes can resolve by alternating truth-values in time, changing logical laws, identifying language or fact conflict and working around it, or choosing correct category level.

logical

Paradoxes {logical paradox} can use faulty laws or misapply logical laws. Logical paradoxes include Epimenides, Russell, Burali-Forti, and relation between two relations that are not so related.

semantic

Paradoxes {semantic paradox} can have ambiguity or error in thought or language. Semantic paradoxes include liar, Berry, Konig {least definable ordinal}, Richard, and Grelling.

Banach-Tarski paradox

Using axiom of choice, fixed-radius Euclidean spheres can map to finite parts that can then make two spheres of same fixed radius {Banach-Tarski paradox}.

barber paradox

A barber says he shaves all those who do not shave themselves and does not shave those who shave themselves, so he can and cannot shave himself {barber paradox}.

Berry paradox

Sets can have the least integer not nameable in fewer than nineteen syllables, but this statement has only 18 syllables {Berry's paradox} {Berry paradox}. However, naming is arbitrary and not the same as nameable.

Chisholm paradox

If surgeons operate, they should use anesthesia. If surgeons do not operate, they should not use anesthesia. Surgeons should operate but do not {Chisholm paradox}.

crocodile paradox

A crocodile tells a parent that he will return a child if parent can guess whether crocodile will return it or not. Parent says that crocodile will not return child {crocodile paradox}.

Fredkin paradox

The more two alternatives are similar, the harder it is to choose, and the less it matters {Fredkin paradox}.

gambler's paradox

For two linked games, at both of which player tends to lose, randomly switching between games can win {gambler's paradox}. One game must have constant event probabilities, and other game must have varying event probabilities. Switching favors keeping gains made, while losses stay constant.

Good Samaritan paradox

If someone regrets crime, person committed the crime {Good Samaritan paradox}. However, people should regret, but people should not commit crime.

heterological paradox

Many words do not describe themselves. Words are heterological if they are not themselves the word. What if the word is the word heterological {heterological paradox}?

lottery paradox

Lotteries have high odds against winning, so no one can believe that they will win. Someone has to win, though nobody can expect to win {lottery paradox}. Therefore, belief probability cannot completely explain belief.

Newcomb paradox

Choosers can select only box 2 or both box 1 and box 2. Box 1 has $1000. Box 2 has $1000000 if predictor predicts chooser will take box 2. Box 2 has $0 if predictor predicts chooser will take both boxes or choose randomly. Predictions have always been correct before. Using expected utility, take box 2, but, using dominance, take both boxes {Newcomb's paradox} {Newcomb paradox}.

preface paradox

Prefaces can state that a book has at least one mistake and that the author stands behind what he or she wrote {preface paradox}. Authors can believe all book statements but also believe that at least one statement is false.

Protagoras paradox

Protagoras gave law lessons to a student who agreed to pay Protagoras after winning a case. The student never got a case, so Protagoras brought the first case against the student, asking specifically for the pay. If student wins case, he both does not and does have to pay {Protagoras paradox}. If Protagoras wins case, he will receive pay, and he will not receive pay.

ravens paradox

Induction can lead to statements but can also lead to statement contrapositives. Contrapositive statements are general, while statements are specific. Evidence for contrapositive statements cannot support statements. For example, "All ravens are black" has support from each raven observation. The statement is logically the same as its contrapositive, "All not black things are not ravens", which also has support from each raven observation {paradox of the ravens} {ravens paradox}.

Russell paradox

A class can be about all things not in the class {Russell paradox} {Russell's paradox}, such as set of all sets that are not members of themselves.

sets

The set of all infinite sets is an infinite set and is a member of itself. The set of all sets is a member of itself. The set of all ideas can be an idea.

However, the set of all men is not a man. Therefore, sets with elements that are not classes cannot be members of themselves.

proof

Assume set of all sets that do not belong to themselves exists and is not a member of itself. Then it must belong to itself by its set definition, causing contradiction. If this set really does belong to itself, then it must not belong to itself by its set definition, causing contradiction. Therefore, set either does or does not belong to itself.

universal set

These two set types are mutually exclusive. The set of all sets that do not belong to themselves cannot be in either of these two set types. Therefore, no universal set exists.

class

Classes {class, set} have sets as members. Classes cannot be class members.

resolution

To resolve the paradox, replace the word "class" with the word "function" in paradox and proof.

Simpson paradox

Probability in combined population can favor one solution, even if probability in separate populations favors another solution {Simpson's paradox} {Simpson paradox}. For example, for population A, solution 1 has probability 2/3 with N = 3, and solution 2 has probability 1/2 with N = 2. For population B, solution 1 has probability 3/4 with N = 4, and solution 2 has probability 5/7 with N = 7. For combined population, solution 1 has probability 5/7 with N = 7, and solution 2 has probability 6/9 with N = 9. Average of population averages is not necessarily combined population average, because some populations have more weight.

mediant fraction

Simpson's paradox follows from mediant fraction properties.

change

If probability of two outcomes varies in one population or set and does not vary in another set, expected outcome can reverse.

MATH>Logic>Paradox>Eubulides

Eubulides paradox

"This statement is false", liar paradox, and masked man paradox {Eubulides paradox} have direct contradiction.

masked man paradox

People know a brother, but, if brother is a masked man, they do not know their brother {masked man paradox}. It depends on opaque reference.

MATH>Logic>Paradox>Identity

identity paradox

One thing is tautologically identical to itself, but two different things cannot be identical {paradox of identity} {identity paradox}. Identity cannot be one relation. Identity requires conjunction of two propositions.

Locke sock

John Locke imagined that he had a sock with a hole. Is a mended sock the same sock {Locke's sock} {Locke sock}?

Theseus paradox

Theseus returned from Crete in a thirty-oared ship. Athenians preserved his ship and, as years passed, replaced planks. Is a ship that has replaced parts the same ship {Theseus' paradox} {Theseus paradox} {Ship of Theseus}? Do things that grow maintain their identity? Is a second ship, built from the old planks, the original ship?

MATH>Logic>Paradox>Liar

liar paradox

People can say that they are stating false statements, but they can be lying, so statements can be both true and not true in all cases {liar paradox}.

Epimenides paradox

This statement is false, or I am lying {Epimenides paradox}.

MATH>Logic>Paradox>Prediction

prediction paradox

An event will happen some day of the next n days, but the day must be such that no one can predict the day {prediction paradox}. Event cannot be on last day, because it is last possible day and so predictable. If it cannot be last day, then it cannot be next-to-last day, because that day has in effect become the last day, and so on, until first day, so event cannot happen.

examination paradox

Examination will be some day next week, but no one can know the day {examination paradox}. Exam cannot be on last day of week, because it is last possible day. Because it cannot be last day, it cannot be next-to-last day, because last day is not possible. It cannot be on other days, because it cannot be on next day.

hangman's paradox

People that are to hang at noon one day of next week cannot predict the day {hangman's paradox}. If seventh day comes and no hanging has happened yet, prediction is possible, so it is clear that they cannot hang on seventh day. Then they cannot hang on sixth day either, and so on, until first day, so no hanging can happen.

MATH>Logic>Paradox>Sorites

sorites principle

Removing small amounts makes little difference, but inconsequential-change series can add to consequential change {sorites principle, logic}.

heap paradox

Heaps of sand are still heaps after removing some grains, but are not heaps after removing too many {heap paradox} {paradox of the heap}. Removing some grains makes little difference, but a series of such inconsequential changes adds to consequential change {sorites principle, heap}. The same idea applies to having hair and being bald {bald man paradox}.

MATH>Logic>Reasoning

reasoning

Thinking {reasoning} can start with true and complete facts and make logically valid inferences. If reasoning needs testimony, testimony must have no bias. All parties accept all judgments. Causal reasoning can have errors. Use effect as cause. Use something as cause just because it happened first. Use merely contributory cause as the only sufficient cause. Use only one cause, when causes are many.

analogy in logic

If two things are alike in some ways, they will be alike in other ways {analogy, reasoning}.

dichotomy in logic

Wholes can divide into two mutually exclusive {disjoint} parts {dichotomy}. Dividing whole into two parts can make non-mutually-exclusive parts. Dividing whole into only two parts can leave out important or necessary third parts.

generalization in logic

Laws can cause generalizations {generalization}. Only laws support counterfactual conditionals. Other generalizations are just situations or accidental generalizations. Laws are inductions from instances, but accidents are not. Laws fit into knowledge systems, but accidents do not.

heuristic reasoning

If statements "if A then B" and "B" are true, then A is probably true {heuristic reasoning, logic}|.

invariance in logic

If premises are invariant under transformation, so is conclusion {invariance, logic}.

property equivalence

Properties are equivalent {equivalence of property} {property equivalence} if they determine same set. The equivalence property sign is double arrow.

sorites reasoning

Reasoning can leave out argument and only give premises and conclusions, if logic follows a recognized syllogism type {sorites, logic}.

square of opposition

Traditional logic used relations {square of opposition} between the four proposition forms to show inferences, contradictions, and contraries.

four forms

All a are b. No a are b. Some a are b. Some a are not b.

contraries

"All a are b" and "No a are b" are contraries, because both can be false and both cannot be true {contrary relation}.

"Some a are b" and "Some a are not b" are subcontraries, because both can be true and both cannot be false {subcontrary relation}.

contradiction

"All a are b" and "Some a are not b" are contradictories, because one must be true and one must be false {contradictory relation}. "No a are b" and "Some a are b" are contradictories, because one must be true and one must be false.

subalternation

"All a are b" entails "Some a are b" {subalternation relation}. "No a are b" entails "Some a are not b".

subalternation

Universal implies particular {subalternation}. "All a are b" entails "Some a are b".

symmetry in logic

If one thing relates to another thing, second thing relates to first thing {symmetry, logic}.

MATH>Logic>Reasoning>Argument

argument in reasoning

Starting from statements, logical steps {argument, logic} can prove that statement is true or false. Arguments link statement and proposition constants and variables. Terms can rearrange or substitute.

variables

Propositions can use variables, such as place and time.

syllogism

Changing verbal argument into syllogism can find inconsistencies and incorrect inferences.

fallacy

Arguments can be invalid, argument forms can be invalid, or proofs can be faulty. Argument irrelevance, invalid deduction, or ambiguity can cause fallacies.

categories

People can make category mistakes.

obligationes

People can state propositions {argument, obligationes}, to which other people {respondent to argument} agree, disagree, or leave open, using relation rules, such as counterfactuals {obligationes}.

argument from authority

Experts or authorities can state that propositions are true or false {argument from authority} {argumentum ad verecundium}, a plausible argument. Showing that proposition is false can show that proposition propounders are not experts or authorities.

argument from ethos

Honest and believable people can state that propositions are true or false {argument from ethos}, a plausible argument. Showing that proposition is false can show that proposition propounders are not honest.

argument from sign

If one property happens, second property happens {argument from sign}, a plausible argument.

argumentum ad ignorantium

People can state that propositions not proved true or false are false or true {argumentum ad ignorantium}, a plausible argument. This relates to burden of proof.

argumentum ad misericordiam

Propositions can have support from emotions, such as pity {argumentum ad misericordiam}, a plausible argument. This appeals to secondary effects.

argumentum ad populum

Propositions can have support by mass opinion {argumentum ad populum}, a plausible argument. This relates to peer pressure, emotional ties, or customs and traditions.

MATH>Logic>Reasoning>Deduction

deduction in reasoning

Starting from true general statement or statements, logical steps prove conclusion true {deduction}. Deduction is true if premises are true.

decision procedure

Proposition proofs have finite numbers of steps {decision procedure}.

existence proof

Proofs {existence proof} can try to show that something exists, preliminary to showing what it is like. Disproving non-existence or proving no non-existence cannot establish existence.

natural deduction

Logic {natural deduction} can have only inference rules, with no axioms. It reaches results but is not about truth. Natural deduction uses sequent calculus. Basic sequent statements are premises or conclusions. Statement sequence shows reasoning chain and conclusion. Introduction rules make more-complex formulas from simpler ones. Elimination rules change complex formulas to simpler formulas. Proofs and truth-trees eliminate formulas by reductio ad absurdum {cut elimination theorem, natural deduction}.

reductio ad absurdum

Proof methods {reductio ad impossibile} {reductio ad absurdum}| {indirect proof} {method of contradiction} {contradiction method} can assume that negative of theorem is true, and then prove that theorem or its premise is false, establishing contradiction. For any component-statement truth-values, contradictions are always false.

MATH>Logic>Reasoning>Induction

induction in logic

Reasoning can go from true similar things to true general thing or pattern {induction, logic}. Starting from examples, induction can formulate conclusion that is not implicit in premises. Properties of some class members can predict properties of all class members.

complete

Premises can be less general than conclusion, but together they can cover all instances in conclusion {complete induction}.

numerical

If property of number one is also property of number n, then property is also property of n+1 and property of all natural numbers {numerical induction}.

eliminative

Observing many examples can find properties that remain constant or true and causes that have effects and can eliminate properties that are untrue or change and causes have no or different effect {Baconian induction} {eliminative induction, Bacon}.

invalid cases

Induction does not always apply. Valid predictions about the future based on hypothesis do not necessarily confirm the hypothesis. Two independent studies can inductively prove hypothesis, but when combined can disprove hypothesis. Highest event probability is not highest combined-event probability. Pairwise probability choices are not necessarily transitive.

mathematical induction

Proof methods {mathematical induction} {first principle of mathematical induction} can be: Prove theorem true for the number one and then, assuming theorem is true for a number, prove it true for any number plus one. Recursive definitions or inductive definitions use mathematical induction.

second induction principle

Proof methods {second induction principle} {second principle of mathematical induction} can be: Theorem can be true for the number one and true for arbitrary number, assuming theorem is true for number minus 1.

MATH>Logic>Rules

absorption rule

If statement implies second statement, first statement implies both itself and second statement: If (p -> q), then p -> (p & q) {absorption rule}.

addition rule

If statement is true, it implies the statement that either the statement is true and/or second statement is true: If p, then (p | q) {addition rule}.

association rule

a & (b & c) = (a & b) & c. a | (b | c) = (a | b) | c {association rule}|.

biconditional

Statements can connect by IF AND ONLY IF ... THEN ..., as in IF AND ONLY IF a THEN b or IFF a THEN b {biconditional} {iff}. If and only if means theorem and converse. Biconditional is true if and only if both statements are true or both statements are false.

commutation rule

a & b = b & a. a | b = b | a {commutation rule}|.

complex constructive dilemma

First statement implies second statement AND third statement implies fourth statement. First statement OR third statement. THEN second statement OR fourth statement {complex constructive dilemma}: (p -> q) & (r -> s). p | r. Therefore, q | s.

complex destructive dilemma

First statement implies second statement AND third statement implies fourth statement. NOT second statement OR NOT fourth statement. THEN NOT first statement OR NOT third statement {complex destructive dilemma}: (p -> q) & (r -> s). ~q | ~s. Therefore, ~p | ~r.

contradiction

No statement can be both true and false {principle of contradiction} {contradiction principle}|.

De Morgan laws logic

In algebra of sets, 1 - (x + y) = (1 - x) (1 - y) and 1 - xy = (1 - x) + (1 - y) {De Morgan's laws, logic} {De Morgan laws, logic}. In propositional logic, not (x and y) equals not x or not y, and not (x or y) equals not x and not y: ~(x + y) = ~x - ~y, and ~(x - y) = ~x + ~y.

deduction theorem

If A1, A2, ..., and An are true, then B is true. A1 is true. A2 is true. ... An-1 is true. If An is true, then B is true {deduction theorem}.

disjunction in logic

Statements can connect by OR (a OR b), where OR is inclusive {inclusive OR, disjunction} {disjunction, logic}| {alternation}. The only false disjunction is if both statements are false. OR can also mean a or b but not both a and b {exclusive OR}.

disjunction rule

If first or second statement is true and second statement is not true, first statement is true: (p | q) & ~p, so q {disjunction rule}|.

distribution rule

a | (b & c) = (a | b) & (a | c) and a & (b | c) = (a & b) | (a & c) {distribution rule}|.

double negation

Negative of a negative statement is statement: ~(~p) = p {double negation}.

excluded middle

Statements are either true or false {excluded middle, principle}| {principle of the excluded middle}. Disjunction of statement and negative statement is true.

exportation rule

The statement that first statement and second statement imply third statement is materially equivalent to the statement that first implies second, which implies third: (p & q) -> r = p -> q -> r {exportation rule}. Exportation is true in propositional calculus. Exportation is not true for strict implication or entailment.

identity in logic

Statements imply that they are true statements {principle of identity} {identity principle}|. If statement, statement is true.

implication

True statements imply true statements {implication, logic rule}|. False statements imply any statement.

material implication

Conclusion is equivalent to negative of conjunction of premise and negative of conclusion {material implication}|: b = -(a & -b). Material implication has sideways horseshoe or arrow -> symbol.

modus ponens

If first statement is true and statement that first statement implies second statement is true, second statement is true {modus ponens, rule}| {detachment rule} {rule of detachment} {affirming the antecedent}. If A is true, and A then B is true, then B is true. p & (p -> q) -> q. Modern formal logic requires only modus-ponens rule.

modus tollens

If A then B is true, and not-B is true, then not-A is true {modus tollens}| {denying the consequent}. If first statement implies second statement is true, and second statement is not true, then first statement is not true: (p -> q) & -q, so -p {principle of modus tollens}.

most statement

Most A is B {most statement} {statement using most}. Most A is C. Therefore, Some B are C.

simple constructive dilemma

First statement implies second statement AND third statement implies second statement. First statement OR third statement. THEN second statement {simple constructive dilemma}: (p -> q) & (r -> q). p | r. Therefore, q.

simple destructive dilemma

First statement implies second statement AND first statement implies third statement. NOT second statement OR NOT third statement. THEN NOT first statement {simple destructive dilemma}: (p -> q) & (p -> s). ~q | ~s. Therefore, ~p.

simplification rule

If the statement "first statement AND second statement" is true, first statement is true {simplification rule}: (p & q), so p.

strict implication

P implies Q if and only if it is impossible that P is true and Q false {strict implication}| {logical implication}: (p -> q) = ~(p & ~q).

transposition rule

If first statement implies second statement, then second-statement negative implies first-statement negative {transposition rule}|: if (p -> q), then (-q -> -p).

MATH>Logic>Statements

statement in logic

Logic can use declarative sentences {statement, logic} {logical statement} with only constants, no variables. Statements are always either true or false.

relations

Statement contains subject and action, state, or relation {predicate}. Statements have. Statement uses basic elements {symbol} with no meaning.

meaning

Statement meaning associates actual objects and events with symbols and relations. For example, statements can describe parts, relations among parts, and spatial axes.

language

Statements are built from pre-language ideas such as number, case, gender, nouns, verbs, modifiers, tenses, gerunds, infinitives, particles, prepositions, and articles. Pre-language ideas include spatial and temporal ideas such as line, group, boundary, figure and background, movement, ascending and descending, association, and attraction and repulsion.

proposition in logic

Logic can use declarative sentences {proposition, logic} {formula, logic} with constants and variables. Propositions are true or false, depending on variable substitution. Propositions contain subject and predicate. Propositions allow analogy, perspective, coordinates, space, time, fields, stories, and images.

undecidability

Propositions can be neither provable nor unprovable {undecidability, logic}, because proof is not finite. Propositional-calculus propositions are decidable. Predicate-calculus theorems have no terminus and are not decidable.

contrary in logic

Two opposing statements cannot be true, but both can be false {contrary, logic}. Two propositions can be contraries, such as "All A are B" and "No A are B". Two opposing statements sometimes are not both false {subcontrary}.

MATH>Logic>Statements>Function

propositional function

Functions {propositional function} can have formulas with variables. Propositional functions are true or false, depending on variable value.

sentential function

Subjects and predicates with variables {sentential function} can make sentences. Sentential functions have constant connectives.

MATH>Logic>Statements>Permutations

contradictory

Premise's negative {contradictory, logic} has opposite truth-value.

contrapositive

Changing subject to negative of predicate and predicate to negative of subject can make new statements {contrapositive}| {transposition, logic} {contraposition}. "If A then B" transposes to "If not B, then not A". If theorem is true, contrapositive is true. If contrapositive is true, theorem is true. "All A are B" transposes to "All not B are not A". "Some A are not B" transposes to "Some B are not A". "No A are B" transposes to "No not B are not A". "Some A are B" transposes to "Some not B are not A". For true statements, contrapositive of "All A are B" and "Some A are not B" are true. For true statements A and B, "No A are B" and "Some A are B" contrapositives are not true.

converse

Exchanging subject and predicate can make new statements {converse, logic}|. For example, "No A is B" has converse "No B is A". "Some A is B" converts to "Some B is A". For categorical statements "No A is B" and "Some A is B", if statement is true, converse is true. All other categorical converse statements must have independent proof. Conversion {conversion per accidens} from "All A are B" to "Some B are A" is valid.

denial in logic

Changing connectives to opposite makes negative statements {denial} {negation}. NOT added to statements denies statement: NOT a or -a. Negation changes statement truth-value. Negative statements only entail similar qualities and are less specific. Negation can result in statements about ambiguous, non-existent, or arbitrary things. In common language, negative particles, word meanings, or inflections negate statements.

equipollence

Negation can apply to sentences with quantities. NOT every a is b, so Every a is NOT b {equipollence, negation}|.

inversion

Changing subject to negative subject {inversion, logic} {inverse, logic} makes new statements. For example, "All A are B" inverts to "All not A are B". "All A are B" implies "Some not A are not B". If inverse is true, converse is true. If converse is true, inverse is true.

obverse

Changing predicate to complement or negative and negating the statement can make new statements {obverse}|. Obversions of the four categorical forms are valid. All A are B, so Not (All A are not B). Some A are B, so Not (Some A are not B). Some A are not B, so Not (Some A are B). No A are B, so Not (No A are not B). If converse is true, obverse is true.

MATH>Logic>Statements>Reasoning

premise

Facts, judgments, or expert testimony can provide statements from which to reason {premise}. Arguments start with premises. Premises can be definitions, axioms, postulates, or previously proved theorems.

not contradictory

Premises must have no contradictions.

not dependent

Premises must be independent.

definition

Definition includes how to use word {operational definition, premise}, what words can substitute {synonym, premise}, class and distinction, pointing at object {ostensive definition, premise}, and conceiving example. Definition can eliminate ambiguity and clarify idea. Word has precise meaning {denotation, meaning}, as well as properties, associations, and feelings {connotation, meaning}. Theory can explain definition, which can influence attitudes and increase vocabulary.

axiom

Premises can be about general or fundamental objects or symbols, assumed to be true.

postulates

Premises can be general statements about mathematical or logical objects and symbols. Anything implied by an elementary and true proposition is true. Disjunction of proposition with itself implies proposition: (a | a) -> a. Propositions imply disjunction of themselves and other propositions: a -> (a | b). Disjunction of one proposition with another implies disjunction of other and first {commutation, postulate}: (a | b) -> (b | a). Disjunction of proposition with disjunction of two other propositions implies disjunction of second with disjunction of first and third {association, postulate}: (a | (b | c)) -> (b | (a | c)). Assertion that statement is true and that the statement implies second statement is true is equivalent to assertion that second statement is true {modus ponens, postulate}. a. a -> b. b. Assertion that proposition implies second proposition is equivalent to disjunction of inverse of first proposition and second proposition {implication, postulate}. (a -> b) = (-a | b). These postulates have no proof that they are independent or consistent.

conclusion in logic

Reasoning wants to obtain true results {conclusion, logic} {theorem}. Conclusions are true or false. Conclusions must be more general than premises.

corollary

If theorem is true, other statements {corollary} can be true.

lemma

To prove theorems can require first proving another theorem {lemma}.

MATH>Logic>Statements>True

analytic statement

Statements {analytic statement} can be true because subject and object meanings are the same, or one is part of other. Using general logical laws and definitions, denying analytic statements leads to contradiction, so analytic statements are logically necessary.

necessary truth

Statements {necessary truth}| can be true and cannot be false, like arithmetical equalities.

tautology

Statements {tautology}| {valid formula} can be true in themselves, without reference to anything else. Tautologies are true for any term or predicate substitutions. Tautologies have necessary and sufficient conditions. Tautologies depend on accepted definitions. The opposite of tautology is self-contradictory.

MATH>Logic>Statements>Kinds

categorical statement

Statements {categorical statement} can use All, Some, or No.

existential import

Statements {existential import} can imply that something exists, as in "Some A Is B". Existential import can be in statements with "Some" and in statements with "All" or "No" that refer to particular or individual.

modal statement

Statements {modal judgment} can be about the possible or necessary.

MATH>Logic>Syllogism

syllogism

Arguments {syllogism} can have general statement or assumption {major premise}, fact or information {minor premise}, and statement to prove {conclusion, syllogism}.

types

Syllogism types correspond to statement types used for premises. Traditional logic used different premise figures and moods to make 16 * 16 = 256 syllogisms. 24 are valid. 19 are strong, because conclusion is as strong as the premises. 15 have equal strength in premises and conclusion {fundamental syllogism}.

Allowing negations, the eight possible forms each can have eight different expressions, making 64 possibilities for each two-premise-one-conclusion combination. Therefore, 64 * 64 * 64 different syllogisms exist.

universals

There cannot be particular solution with two universal premises.

alternative syllogism

Syllogisms {alternative syllogism} can use OR. Alternative syllogism has the following forms. Either A or B is true, not A is true, so B is true. Either A or B is true, not B is true, so A is true. A and B do not have to be mutually exclusive.

antilogism

Syllogisms {antilogism} can have two premises and conclusion negation. Statement pairs can lead to third-proposition negation.

categorical syllogism

Syllogisms {categorical syllogism} can have major premise, minor premise, and conclusion. Major premise and minor premise share term. For example, "all a are b" is major premise. "c is a" is minor premise. Therefore, "c is b" {conclusion}, because both premises share term a.

negative

Categorical syllogism can have only one negative premise, and then conclusion must be negative.

quantifier

Categorical syllogisms can have premises about all, some, or none, in four forms: "All A are B", "No A are B", "Some A are B", or "Some A are not B". Verb must be "to be".

First, statement states All, Some, or No {quantifier, syllogism}. Noun phrase {subject class} follows quantifier. Verb {copula, syllogism} follows subject class. Second noun phrase {predicate class} follows copula. Therefore, categorical syllogism has three terms: two noun phrases and copula.

types

Categorical syllogisms have four types {figure, syllogism}. All A are B, C is A, and so C is B {first figure of syllogism}. All B are A, C is A, and so C is B {second figure of syllogism}. All B are A, A is C, and so C is B {third figure of syllogism}. All A are B, A is C, and so C is B {fourth figure of syllogism}.

The four types can use "All", "Some", or "No", making twelve categorical syllogisms. With "Some" in conclusion {particular statement}, only one premise can have "All" or "No" {universal statement}.

errors

Errors can be in categorical syllogisms. Major premise can be untrue. Syllogism can reverse major premise. Terms can be ambiguous. Middle term can be ambiguous {ambiguous middle}. For example, A is B(1) is true, B(2) is C is true, so A is C is true.

Celarent Barbara

Hexameter rhymes {Celarent Barbara} [1200] can be mnemonics for valid syllogisms.

conditional syllogism

Syllogisms {conditional syllogism} {hypothetical syllogism} can use IF ... THEN .... Conditional syllogism has the following forms. If A then B is true, A is true, so B is true. If A then B is true, not B is true, so not A is true. Unless A then B is true, not A is true, so B is true. If not A then B is true, not A is true, so B is true. If first statement implies second and second implies third, first implies third: if (p -> q) & (q -> r), then (p -> r).

disjunctive syllogism

Syllogisms {disjunctive syllogism} can use NOT BOTH. Disjunctive syllogism has the following forms. Not both A and B is true, A is true, and so not B is true. Not both A and B is true, B is true, and so not A is true.

distributed middle

Middle term must distribute at least once in categorical syllogism {distributed middle}|.

distribution in logic

Statements can exclude or include all members {distribution, logic}. Distributed terms refer to all individuals. In "All A are B", A distributes, but B does not. In "No A are B", A and B distribute. In "Some A are B", A and B do not distribute. In "Some A are not B", A does not distribute, but B distributes. Categorical syllogisms have three terms. If one term distributes in premises and either of the other two terms does not distribute in premises, the other two terms cannot distribute in conclusion.

MATH>Logic>Kinds

closed logic

Logic {closed logic} can use intersection and account for all intersection cases.

complementary logic

Complementary propositions have no simultaneous decidability of incompatible or non-commuting propositions {complementary logic}.

fuzzy logic

Logic {fuzzy set} {fuzzy logic} can use probability.

modal logic

Whenever A entails B, it must be impossible for A to be true without B also being true {modal logic, relevance} {relevance logic}. In modal-logic predicate calculus, Necessarily and Possibly are constants.

non-distributive logic

Distributive law does not hold in some logics {non-distributive logic}.

non-monotonic logic

Adding new premise to system can make previously valid conclusions become invalid {non-monotonic logic}.

ontology logic

Logic {ontology logic} can use abstract quantifiers.

probability logic

Truth-value can have probability {probability logic}.

protothetic logic

Logic {protothetic logic} can use equivalence.

quantum logic

Quantum mechanics violates regular logic, because proposition "A and B" can be false even if both A and B are true {quantum logic}. Quantum logic can make logic relative.

symbolic logic

& means AND {symbolic logic}. | means OR. - means NOT. Parentheses delimit compound or complex statements. Implication, or other logical operation, needs no other symbol.

three-valued logic

Statements about future contingent events are neither true nor false, requiring logic with three values {three-valued logic}.