Outline of Geometry
September 5, 2013

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Note: To look up references, see the Consciousness Bibliography, listing 10,000 books and articles, with full journal and author names, available in text and PDF file formats at http://www.outline-of-knowledge.info/Consciousness_Bibliography/index.html.

MATH>Geometry

geometry in mathematics

Mathematics {geometry} can study space properties, such as distances and angles. Historically, geometry developed from techniques to measure land, construct temples, and observe stars.

concurrent figures

Several figures or lines {concurrent figures} can share points.

Erlanger program

Geometry can have transformation groups that have invariants {Erlanger program}. Possible invariants are linearity, collinearity, cross ratio, harmonic point set, and conic section. According to the Erlanger Programme [1872] (Felix Klein), geometry is point spaces and their transformation groups (mappings). For a geometry, specific invariant space structures do not change under its transformations. For Euclidean geometry, rigid motions (displacement and rotation transformations) do not change invariant separation. Euclidean geometry has between-ness and separation (length). For similarity or extended Euclidean geometry, rigid motions, translations, rotations, and uniform scalings (similarity transformations) do not change invariant length ratios. Extended Euclidean geometry has between-ness, length ratios, and angles. For affine geometry, translations, rotations, skewings, non-isotropic scalings, and nonsingular linear mappings (affine transformations) do not change invariant line at infinity or three-collinear-point cross ratio. Affine geometry has between-ness but no length ratios. For projective geometry, all linear mappings (projective collineations) do not change invariant four-collinear-point cross ratio. Projective geometry has no separation, length ratios, or between-ness. Euclidean geometry has the most invariants, and projective geometry has the least.

locus of points

Points, lines, or curves can trace paths {locus of points}, using conditions, equations, or inequalities.

nomogram

Graphics {nomogram} can contain three scales. If two scales have known values, straightedge can determine value on third scale.

sangaku

Japanese temple geometry {sangaku}.

MATH>Geometry>Cartography

cartography

Geometry includes making maps {cartography}.

notation of maps

Symbols {notation, map}, such as colors, contours, and dots, describe maps.

projection for map

Drawing maps can use projective metric geometry {projection, map} to put actual surface onto map.

scale of map

Map relates map distances to actual distances {scale, map}. Scales use units or ratios. For example, rulers can be one meter long, marked in centimeter units.

topographic map in cartography

Maps {topographic map, cartography}| can have both natural features and human-made features.

MATH>Geometry>Construction

construction geometry

Drawing figures {construction, geometry} can use only ruler and compass.

geometric proof

To prove using geometry {geometric proof}, use only movable and changeable angle {compass, geometry} and straight line {straightedge}.

trisection

Straightedge and compass can divide line segments into three equal parts {trisection}.

MATH>Geometry>Coordinate System

bipolar coordinate

Points in planes can have distances {bipolar coordinate} to two fixed points.

Cartesian coordinate

Points in planes have distances to x-axis {abscissa} and distances to y-axis {ordinate} {Cartesian coordinate}. Cartesian coordinates use perpendicular axes {rectangular coordinate}. x-axis and y-axis divide the plane into four parts {quadrant, plane}. Axes intersect at point {origin}. In Cartesian coordinates, distance between two points is ((x2 - x1)^2 + (y2 - y1)^2)^0.5. Space points have distances to x-axis, y-axis, and z-axis. x-axis, y-axis, and z-axis are perpendicular.

homogeneous coordinate

System, such as Cartesian space, can have three perpendicular axes {homogeneous coordinate}. Point has coordinates that are perpendicular distances to axes. For homogeneous coordinates u, v, and w {line coordinate}, u*x + v*y + w*z = 0 represents a line.

Using homogeneous coordinates, all curve equations are first-degree homogeneous equations. Equation degree gives curve class. For example, quadratic equations are surfaces.

triangle

In triangle, point has signed perpendicular distances to triangle sides. If triangle side is at infinity, x = x1/x3 and y = x2/x3.

comparison

For point coordinates, equation degree is curve order.

duality

Line coordinates prove the duality principle that points and lines are complementary.

polar coordinate

Plane can use radius and angle to x-axis as coordinates {polar coordinate}, rather than x and y coordinates.

pole

Origin is reference point {pole, coordinate}.

polar axis

x-axis is reference line {polar axis, coordinate}.

coordinates

Plane points have distance to pole {radius, coordinate} and angle to polar axis.

straight line equation

Straight line has equation r * cos(A) = b, where r is radius, A is angle to polar axis, and b is constant. Straight line can have equation r * cos(a - A) = b, where a is angle to axis perpendicular to polar axis {angle of inclination} {inclination angle}. Slope is tan(a).

circle equation

Circle has equation r^2 - 2 * r * c * cos(A) + c^2 = d^2, where r is radius, A is angle to polar axis, and a, c, and d are constants.

parabola equation

Parabola has equation r = (e * a) / (1 - e * cos(A)), where e is eccentricity and a is constant.

hyperbola equation

With polar coordinates centered at a focus, hyperbola has equation r = (e * a) / (1 + e * sin(A)), where e is eccentricity and a is constant.

ellipse equation

Ellipse has equation r = (e * a) / (1 - e * sin(A)), where e is eccentricity and a is constant.

spiral equation

Polar equations r = A * a, where a is constant, graph to spirals {Archimedes spiral}.

rectangular coordinates

Polar coordinates relate to rectangular coordinates. x = r * cos(A), y = r * sin(A), r = (x^2 + y^2)^0.5, and tan(A) = y/x.

cylindrical coordinates

Space points can use plane polar coordinates and reference line perpendicular to the plane from pole {cylindrical coordinate}. Points have distance to pole, distance along perpendicular axis, and angle to polar axis. Cylindrical coordinates relate to rectangular coordinates. x = p * cos(A), y = p * sin(A), and z = z.

spherical coordinates

Space points can use pole and two perpendicular reference lines through pole {spherical coordinate}. Points have radius to pole and two angles to the reference lines. Spherical coordinates relate to rectangular coordinates. x = r * sin(A) * cos(B), y = r * sin(A) * sin(B), and z = r * cos(B). Spherical coordinates relate to cylindrical coordinates. p = r * sin(A), z = r * cos(A), r = (p^2 + z^2)^0.5, and A = arctan(p/z).

MATH>Geometry>Fractal Geometry

fractal curve

Curves {fractal curve}| can have non-integral dimension.

dimension

Dimension d depends on unit-copy number m needed to make shape that number n of times bigger: $m = n^d$. For example, line segments can be two times longer using two line segments, so dimension is 1: $2 = 2^1$.

If fractal unit has _/|_ shape, next larger self-similar shape is _/|_, and line segments look like original unit. The next-larger shape is three times bigger and needs four unit copies, making dimension 1.26186...: $4 = 3^{1.26186...}$

self-similarity

For fractals, whole shape is similar to part shape {self-symmetry, polar} {self-similarity}. Scale changes do not change pattern. Fractals can model objects that have same shape at different scales. Fractals can model renormalization.

non-linear

Fractals are non-linear.

fractal limits

Fractal shape is the limit of iteratively applying mapping rules.

rule

Given fractal shapes, using same shape at smallest scale can induce rules for making the shape {collage theorem}.

examples: Mandelbrot curve

Fractal curves {Mandelbrot curve} can enclose finite or zero area but have infinite length. Infinite length fills two-dimensional space. Fractal curve has physical dimension 1. If fractal curve fills two-dimensional plane, it has fractal dimension 2.

examples: Peano curve

Fractals can be curves {Peano curve}.

examples: Koch curve

Starting with a triangle, repeatedly adding triangle one-third the size to line-segment middles makes curves {Koch curve}. Boundary has infinite length but finite area.

examples: Sierpinski carpet

Starting with square, making nine squares inside, removing central square, and then repeating makes surfaces {Sierpinski carpet}.

examples: Sierpinski gasket

Starting with equilateral triangle, making nine equilateral triangles inside, removing center equilateral triangle, and then repeating makes surfaces {Sierpinski gasket}.

examples: Menger sponge

Menger sponges are three-dimensional Sierpinski carpets.

examples: nature

Natural fractals are coastlines, rivers, islands, seas, lakes, mountains, arteries, music, Brownian motion paths, critical points, elasticity, turbulence, snowflakes, clouds, disconnected star-cluster points, temperature, spectra, and all intensive properties.

uses

Relief maps, Cantor infinite sets, computer designs, and Fourier analysis can use fractals.

fractal interval

Intervals {fractal interval} can be harmonic, as in logarithmic relations, instead of linear or vectorial. For example, remove interval middle third, then remove middle thirds of both remaining line segments, and so on, indefinitely, to make triadic set. Intervals can use other numbers, proportions, sizes, and positions of such cutouts. Cutouts can be random or fixed.

purposes

Triadic sets, and similar point distributions over intervals, model hierarchical errors, time-measurement errors, high signal-to-noise-ratio noise, negligible thermal noise {excess noise}, computing errors, and other errors in which, as time goes up, error chance goes down.

dimensions and fractals

Points have dimension zero. Sets of countable separated points have dimension zero. Sets of uncountable separated points have no density and have topological and fractal dimension zero. Line segments are point sets with linear density and have topological and fractal dimension one. Bounded surfaces are point sets with surface density and have topological and fractal dimension two. Bounded volumes are point sets with volume density and have topological and fractal dimension three.

Fractals are geometric figures with non-integer dimensions. Some geometric fractals start with a line segment and repeatedly remove intervals. Repeatedly removing intervals (to make Cantor sets, for example) keeps topological dimension one but reduces fractal dimension to less than one.

Some geometric fractals start with a line segment and repeatedly replace intervals with added values. Repeatedly replacing intervals with added values (to make Koch curves, for example) makes topological dimension two and fractal dimension greater than one.

To make more than one dimension, fractals use complex numbers, which have two components and so can graph to surfaces. Some geometric fractals start with a bounded surface and repeatedly remove inner regions, to make topological dimension two and fractal dimension less than two. Some geometric fractals start with a bounded surface and repeatedly replace inner regions with added values, to make topological dimension three and fractal dimension greater than two.

To make more than two dimensions, fractals use hypercomplex numbers, which have three or more components and so can graph to volumes and hypervolumes. Some geometric fractals start with a bounded volume and repeatedly remove inner regions, to make topological dimension three and fractal dimension less than three. Some geometric fractals start with a bounded volume and repeatedly replace inner regions with added values, to make topological dimension four and fractal dimension greater than three.

Levy flight

Generalized Brownian motion {Levy flight} has different-length jumps in all directions. Levy flights are isotropic. Jump-length distribution has fractal dimension.

self-symmetry

If fractal-curve or figure scale changes, curve has same shape {self-symmetry, fractal}| {fractal hierarchy}. Whole shape is similar to part shape.

MATH>Geometry>Fractal Geometry>Fractal Set

fractal set

Sets {fractal set}| can have members related by fractal steps. All initial values used in Newton's method lead to square root. Following fractal paths, initial value lying between two roots can lead to any value. Feigenbaum functions on complex plane reach all frequencies {Julia set}. Julia sets can generalize. Take complex number, square it, add square to original complex number, square result, add square to original complex number, and so on. If result does not diverge from complex-plane origin, result has bound, and complex number is in set {Mandelbrot set}. If real or imaginary part becomes greater than 2 or smaller than -2, number diverges. Mandelbrot set is boundary between the regions {fractal basin boundary}. Mandelbrot set is not precisely self-similar.

Levy set

Sets {Levy set}, such as rectifiable or connected curves, can have dimension between zero and one and density between zero and one. Points have dimension zero. Lines have dimension one. Lower dimension means more clustering.

triadic set

Remove line-segment middle third, then remove middle thirds of both remaining line segments, and repeat indefinitely {triadic set}.

MATH>Geometry>Mapping

mapping in geometry

Ordered-set points can correspond to other ordered-set points {mapping, geometry}.

congruence

Figures can have same line segment lengths and angles {congruence}|.

homothetic ratio

Lines passing through corresponding points of a figure set can meet at a fixed point {center of homothety} {center of similitude} {similitude center} {ray center} {homothetic center} {radially related figures}. Corresponding-point distances from ray center have constant ratio {homothetic ratio} {ray ratio} {ratio of homothety} {ratio of similitude} {similitude ratio}.

intercept in geometry

Two configurations can coincide at point {intercept, geometry}.

monotonic mapping

Ordered set can map onto ordered set so element preceding or equaling a first-set element has image that precedes or equals image of a second-set element {monotonic mapping}. Monotonic mapping preserves order.

similar figure

Figures can have constant ratio between corresponding line segments {similar figure}.

similar surface

Corresponding-point pairs can have constant ratio between distances {similar surface}.

similitude

Similar figures can be radially related {similitude}.

superimposable figure

Figures {superimposable figure} can be congruent.

MATH>Geometry>Plane

plane geometry

On planes, straight lines intersect at one or no points {plane geometry}|. Two points determine a line. For every straight line, through any point outside the line, one line is parallel to the line. In triangles, angle sum is 180 degrees.

plane as surface

Flat surfaces {plane, geometry} have two dimensions.

area

Surfaces have extent measures {area, surface} {surface area}.

MATH>Geometry>Plane>Angle

angle in geometry

The point at which two lines intersect has opening {angle, geometry} between lines.

bisector

Lines {bisector, angle}| can divide angles in half.

half-line

Angles have sides {half-line} {ray, angle}.

internal bisector

Bisectors {internal bisector} can be perpendicular to external bisector.

nodal point in tessellation

In tessellations, vertexes {nodal point, tessellation} can be common to three or more polygons.

MATH>Geometry>Plane>Angle>Size

degree of angle

Circles can divide into 360 equal arcs {degree, angle}. Degrees can divide into 60 parts {minute, angle}. Minutes can divide into 60 parts {second, angle}.

grad

Angle units {grad} {grade, angle} can equal 1/100 right angle.

perigon
2*pi radians, or 360 degrees, make one revolution {perigon}.

radian
Circles with one-unit-length radius have circumference 2*pi radians, so 360 degrees equals 2*pi radians {radian, angle}. One radian equals 360 / (2 * pi) degrees or 57 degrees.

MATH>Geometry>Plane>Angle>Kinds

acute angle
Angles {acute angle}| can be less than 90 degrees.

complementary angle
Two angles {complementary angle}| can add to 90 degrees.

corresponding angle
Two angles {corresponding angle, plane} can have parallel corresponding sides.

directed angle
One ray {initial side} can be stationary and one ray {terminal side} can rotate, so angle {directed angle} changes.

equal angle
Two angles {equal angle} with parallel sides are equal.

exterior angle
Polygon sides make angles {exterior angle} outside polygon vertices. External angle is 360 degrees minus internal angle.

interior angle
Two polygon sides make an angle {interior angle} {internal angle} inside a polygon vertex.

obtuse angle
Angles {obtuse angle}| can be between 90 and 180 degrees.

quadrantal angle
Angles {quadrantal angle} can be (n * pi) / 2 radians, where n = 0, 1, 2, ..., for 0, 90, 180, ... degrees.

reflex angle
Angles {reflex angle, size} can be greater than pi radians and less than two times pi radians.

straight angle
Angles {straight angle} can be pi radians.

supplementary angle
Two angles {supplementary angle}| can add to 180 degrees.

vertical angle
Two lines intersect to form opposite angles {vertical angle}.

MATH>Geometry>Plane>Circle

circle in geometry
Closed geometric figures {circle, geometry} have centers and circumferences. Circle equation is (x - h)^2 + (y - k)^2 = r^2, where r is radius. For given perimeters, out of all plane figures, circles bound greatest area {area, circle}. Circle area = pi * r^2, where r is radius.

annulus

Plane figures {annulus}| can have ring shape.

arc of circle

Circles can have parts {arc, circle} greater than semicircles {major arc} or less than semicircles {minor arc}. Arcs subtend central angle. Area subtended by circle arc = 0.5 * r^2 * A, where A is angle in radians, and r is radius. Arc length s equals radius r times angle A in radians: s = r*A. In two different circles, for same angle, arc-length ratio is proportional to radii ratio.

central angle

Circles can have angles {central angle} between two radii.

chord of circle

Line segments {chord, circle}| can link two circle points. Diameter is longest chord.

circular measure

2*pi radians equals 360 degrees {circular measure}. pi radians equals 180 degrees. pi/2 radians equals 90 degrees.

circumference

distance around circle {circumference}.

curvature of circle

Circles have radius-length reciprocal {curvature, circle}.

inverse curve

All curve points can invert {inverse curve}. Operations {inversion, curve} can find inverse curves.

long radius

For polygons, circumscribed-circle radius {long radius} is longer than inscribed-circle radius.

perimeter of circle

Circle perimeter is 2 * pi * radius {perimeter, circle}.

Ptolemy theorem

Circles can have inscribed quadrilaterals, which have two diagonals. Diagonal product equals sum of opposite-side products {Ptolemy's theorem} {Ptolemy theorem}.

quadrant of circle

Circle regions {quadrant, circle}| can include one-quarter circumference and two radii.

rectangular properties

Two intersecting lines intersect a circle to make line segments {rectangular properties}.

secant circle

Chords {secant, circle}| can extend beyond circles.

sector circle

Circular arc and radii from endpoints make pie-piece figures {sector, circle}| {segment, circle}. Sectors {major sector, circle} {major segment, circle} can be greater than semicircles. Sectors {minor sector, circle} {minor segment, circle} can be less than semicircles.

semicircle

Diameter ends define half circle {semicircle}|. Angle from circle point to diameter ends is right angle.

subtend

Arcs define {subtend}| central angle. Area subtended by arc is 0.5 * r^2 * A, where r is radius and A is arc length in radians. Arc length s equals radius r times angle A in radians: s = r*A. In two different circles, for same angle, arc-length ratio is proportional to radii ratio.

MATH>Geometry>Plane>Circle>Generated Solids

band of sphere
Solids {zone, sphere} {band}| can result when circle sector rotates around sphere diameter that does not pass through sector. Zone is on sphere surface. Right-circular cone connects sphere center to closer zone base.

cap using sector
Solids {cap, circle}| {cap zone} can result when circle sector rotates around sphere diameter that passes through sector. Right-circular cone connects sphere center to cap base.

spheroid
Circles can make ellipsoids {spheroid} of revolution.

MATH>Geometry>Plane>Circle>Point

circular point
Two imaginary points {circular point at infinity} on line at infinity are common to two circles.

concyclic point
Several points {concyclic point} can lie on same circle.

power of point
Line from point intersects circle at two points, to form secant. Distances from point to both points can multiply {power, point}. Point power is negative if point is inside circle and positive if point is outside circle.

MATH>Geometry>Plane>Circle>Kinds

inversion circle
For circles, radius midpoints define smaller-circle {inversion circle} centers that intersect first circle at only one point and include larger-circle center. Radius diameters intersect first circles at points {inverse point}. Distance from first intersection to first-circle center times distance from inverse point to first-circle center equals r^2 {inversion constant}.

concentric circle
Circles {concentric circle} can have same center.

escribed circle
Circles {escribed circle} can touch three consecutive polygon sides, if two polygon sides extend.

great circle
Circles {great circle}| on spheres can have same radius as sphere.

imaginary circle
Equation (x - a)^2 + (y - b)^2 + c^2 = 0 has radius = i*c {imaginary circle}.

inscribed circle
Circle can touch three consecutive polygon sides {incircle} {inscribed circle}. Inscribed circle has center {incenter}.

orthogonal circle
Two circles {orthogonal circle} can intersect at right angles. Curves {orthogonal trajectory} can intersect all curve-family members at right angles.

small circle
Plane intersects sphere to make circle {small circle}. Small circle does not include great circle.

MATH>Geometry>Plane>Curve

curve of line
Line segments can be straight or not-straight {curve}.

arc of curve
Curves have parts {arc, curve}.

asymptote of curve
At large positive or negative values, curves can approach straight lines {asymptote, curve}.

family of curves
Equations with parameters can define sets of curves {family of curves}.

Peaucellier linkage
Linkages {Peaucellier's linkage} {Peaucellier linkage} can construct inverse curves.

perimeter of figure
Figures have length around edges {perimeter, curve}.

rectification of curve
Curved line-segment length {rectification of curve}| is integral from x = x1 to x = x2 of (1 + (dy/dx)^2)^0.5 * dx.

rolling motion
Moving curves or surfaces can have contact points or lines with fixed curves or surfaces, with no slippage {rolling motion}. Moving curve rotates around centroid. Centroid revolves around fixed-surface curvature center at contact point or line.

secant curve
Straight lines can intersect curves to form line segments {secant, curve}|.

segment of curve
Curve parts {segment, curve} lie between two points. Area between arc and chord makes segment. Solid formed by two parallel planes intersecting sphere makes segment.

subnormal of curve
Normals at curve points can make orthogonal projections {subnormal, curve} onto x-axis.

subtangent
Tangents at curve points can make orthogonal projections {subtangent} onto x-axis.

supplemental chord
Chords {supplemental chord} can go from any circle or ellipse point to diameter end.

MATH>Geometry>Plane>Curve>Point

acnode
Function points {acnode} {isolated point} can have no nearby points that belong to function. Acnodes must be at least double points.

centroid
Points {centroid, curve} can be curve-point-coordinate arithmetic means.

multiple point
Curve branches meet at a point {multiple point}.

node of curve
Two curve branches can meet at point {node, curve} {double point}. Both branches can have real and distinct tangents {crunode}. Both branches can have real and coincident tangents {cusp, curve}. Both branches can have imaginary tangents at acnodes.

osculation point
Curves can have double cusps {tacnode} {point of osculation} {osculation point}.

stationary point
Turning points or maximum or minimum points {stationary point} do not have to be inflection points.

MATH>Geometry>Plane>Curve>Kinds

brachistochrone
Wires can have a shape {brachistochrone} so that a bead can slide from one end to the other in shortest possible time.

exponential curve
Equation y = a * e^(b*x) defines curve {exponential curve}|.

glissette
Points or envelopes can slide along two fixed curves and make curve {glissette}.

integral curve
Curve families {integral curve} can be differential-equation solutions.

isoclinal
For successive c values, function f(x,y) = c makes curves {isoclinal}. If dy/dx = f(x,y), solutions have graphs.

Jordan curve
Lines {Jordan curve} can have no multiple points. Plane closed curves have inside and outside {Jordan curve theorem}.

parallel curves
Two curves {parallel curves} can have same normal, have same curvature center, and be in one-to-one correspondence.

Peano-Hilbert curve
Continuous curves {Peano-Hilbert curve} can fill continuous two-dimensional regions. Start with square. Put four squares inside and connect centers to make polygonal curve. Put four squares inside small squares to make 16 squares and connect centers to make polygonal curve. Continue to make polygonal curve that approaches passing through all points inside starting square.

quadratrix curve
Hippias of Elias [-430 to -420] invented a curve {quadratrix curve} {trisectrix}. Begin with semicircle and line segment equal to radius but tangent to semicircle at end. Move line segment through semicircle uniformly, keeping direction, until it is tangent to other semicircle end. Simultaneously rotate ray, starting from same semicircle end as line segment, around semicircle until it goes through other semicircle end. Line segment and ray intersect to form a curve.

quadric curve
Curves {quadric curve} can have second-order equations. Surfaces {quadric surface} can have second-order equations.

Riemann-Weierstrass curve

Continuous curves {Riemann-Weierstrass curve} can have no direction at all points. Start with straight line segment with slope +1 and straight line segment with slope -1, meeting in middle, like this: ^. Then, on line segments, repeat construction, to make shape like M, with four line segments, placing base endpoints half as far apart as before. If repeated, both endpoints approach same point, and slopes approach infinity.

sine curve

Curves {sine curve} can look like waves.

sinusoid

Curves {sinusoid} {sinusoidal curve} can be like sine curves.

tangent curve

Curves {tangent curve} can have same tangent at points.

tangent graph

Graphs {tangent graph} can look like third-power functions.

trisectrix of Maclaurin

x^3 + x * y^2 + a * y^2 - 3 * a * x^2 = 0 defines curves {trisectrix of Maclaurin}, which can trisect angles.

unicursal curve

One continuous xy-plane curve {unicursal curve, continuous} can make x and y be finite continuous functions of a parameter.

MATH>Geometry>Plane>Curve>Kinds>Envelope

envelope

Curves {envelope, curve}| can be tangential to all curve-family curves. Curve families can have equation f(x,y,a) = 0, for parameter a. Curve families have boundary curves. Eliminating parameter from function and taking partial differential with respect to parameter can find envelope.

evolute

Curve normals {evolute} can form envelopes. Curve evolute is curvature-center locus.

MATH>Geometry>Plane>Curve>Kinds>Rolling

cardioid

Epicycloids {cardioid} can be curves traced by one circle point rolling on fixed equal-radius-circle outside: r = 2 * a * (l - cos(A)), where r is distance from pole, a is fixed-circle radius, pole is where rolling point meets fixed circle, and A is angle to radius. Cardioids have one loop and are special limaçon-curve cases.

cycloid

Circle points rolling along straight lines make curves {cycloid}. Circle points rolling on fixed-circle outsides make curves {epicycloid}. Circle points rolling on fixed-closed-curve outsides make curves {pericycloid}. Circle points rolling on fixed-circle insides make curves {hypocycloid}. Fixed circles can have four times rolling-circle diameter {astroid}.

limacon of Pascal

Conchoids {limaçon of Pascal} can have circle for fixed curve: r = 2 * a * cos(A) + b, where r is distance from pole, pole is where rolling point meets fixed circle, A is angle to radius, and a and b are constants. If b < 2*a, limaçon of Pascal has two loops. If b > 2*a, limaçon of Pascal has one loop. If b = 2*a, limaçon of Pascal is a cardioid curve.

roulette curve

Curve points or line envelopes rolling along fixed curves make curves {roulette curve}.

MATH>Geometry>Plane>Curve>Kinds>Set

Cassini oval

Oval points can have distances to two fixed points. Product of distances can be constant {Cassini oval}. Product equals b^2, where b is distance when oval point is equidistant from fixed points.

cissoid

Secant endpoints can make curves {cissoid}.

conchoid

Two points on the line passing through fixed point and intersecting fixed curve both maintain equal and constant distance from intersection point and make two curves {conchoid}. Fixed curve is asymptotic to both conchoid branches.

conchoid of Niomedes

Conchoids {conchoid of Niomedes} can have straight lines as fixed curves. Conchoid of Niomedes can trisect angle and duplicate cube.

involute

Points on flexible but not stretchable thread, kept taut while being wound or unwound on another curve, can trace a curve {involute}. Curve used for winding is traced-curve involute.

lemniscate curve

For Cassini ovals, distance between both points can be constant {lemniscate curve}.

lemniscate of Bernoulli

Feet of perpendiculars from rectangular-hyperbola center to all tangents makes a curve {lemniscate of Bernoulli}. If fixed point is circle radius times square root of two from circle center, and two points are secant-through-fixed-point chord length from fixed point, both point loci make lemniscate of Bernoulli.

orthotomic curve

Point maintaining same distance to intersection point for all tangents to fixed curve makes curve {orthotomic curve}.

pedal curve

Points where perpendiculars from fixed points meet tangent to fixed curve point {pole, pedal curve} make curves {pedal curve}. For parabolas, pedal curves are tangents at vertices. For ellipses or hyperbolas, pedal curves are auxiliary circles. In pedal curves, perpendicular length relates to length from fixed point to curve point {tangent-polar equation} {pedal equation}.

spiral

Point sets {spiral} around a fixed point {center, spiral} can have distance {radius vector, spiral} from center to fixed point that relates to rotation angle {vectorial angle}.

equiangular

Spirals {equiangular spiral} can have equal inclinations of radius vector and tangent vector at all points.

types

Archimedes spiral is r = a*A, where a is constant, A is angle, and r is radius. Spirals {parabolic spiral} {Fermat's spiral} can be r^2 = a*A.

Equiangular spirals {logarithmic spiral} {logistic spiral} can be log(r) = a*A.

reciprocal

Spirals {hyperbolic spiral} can be: r*A = a. Hyperbolic spirals are same as inverse {reciprocal spiral}.

loxodromic spiral

Spirals {loxodromic spiral} {rhumb line, spiral} in spheres can cut meridians at constant angle.

strophoid

Straight lines can pass through fixed points {pole, strophoid} to intersect all fixed curve points. Two line points maintain same distance to intersection as distance from intersection to another fixed point, not necessarily on line, to make a curve {strophoid}.

tractrix

Curve points can have distance to a fixed point. Fixed point has distance from coordinate origin. Curves {tractrix} can have constant ratio of distance from curve point to fixed point to distance from origin to fixed point.

versed sine curve

For radius-a circle with center at (0,a), a straight line from the origin intersects the circle to define the y-coordinate for all x {witch of Agnesi} {Agnesi witch} {versed sine curve} {versiera} (Maria Agnesi): y = 8 * a^3 / (x^2 + 4 * a^2).

MATH>Geometry>Plane>Curve>Kinds>Logistic

logistic curve

Curves {logistic curve} can model growth with growth factors and limiting resources. Growth can depend on growth factors, which can have weights. Limiting resources {bottleneck, growth} can slow growth.

Amount or percentage P at time t is P(t) = A * (1 + m * e^(-t/T)) / (1 + n * e^(-t/T)), where A = constant growth-factor weight, m = original growth-factor amount, n = original limiting-resource amount, and T = time period or inverse growth rate.

Growth is percentage of factor to resource. Denominator and numerator decrease at same rate.

beginning

Original amount depends on relative m and n amounts and on weight A: P(0) = A * (1 + m) / (1 + n). If n is much greater than m and A, denominator is large, and P is zero.

process

At first, growth is exponential with time. Then growth passes through time when growth has constant rate and is linear. Then growth slows exponentially to zero. Amount is then maximum or 100 percent at maturity.

shape

Logistic curve has sigmoid shape.

comparison

Logistic function inverts natural logit function.

logit curve

For numbers between 0 and 1, functions {logit curve} can be logit(p) = log(p / (1 - p)) = log(p) - log(1 - p). Logarithmic base must be greater than 1, for example, 2 or e. p / (1 - p) is the odds, so logit is logarithm of odds. Logistic function inverts natural logit function. Logit functions can be linear {logit model}: logit(p) = m*x + b. Logit models {logistic regression} can be for linear regression.

probit curve

Inverse cumulative distribution functions, or normal-distribution quantile functions, depend on error-function inverses {probit curve}. It changes probability into function over real numbers: probit(p) = 2^0.5 * erf^-(2*p - 1). Probit functions can be linear over a large real-number middle range {probit model}.

sigmoid curve

Logistic curve can look like S {sigmoid curve}| {standard logistic function}. Sigmoid curve starts at minimum or maximum, always increases or decreases, and ends at maximum or minimum. It has one inflection point, near which it grows linearly. It changes exponentially at beginning and end. Amount or percentage P over time t is P(t) = A * (1 + m * e^(-t/T)) / (1 + n * e^(-t/T)). If A = 1, m = 0, n = 1, and T = 1, P(t) = 1 / (1 + e^-t). dP/dt = P * (1 - P), with P(0) = 1/2 and dP(0) / dt = 1/4.

Arctangent, hyperbolic tangent, and error function make sigmoid curves.

double sigmoid curve

Curves {double sigmoid curve} can look like double S. Double sigmoid curves start at minimum or maximum, always increase or decrease, and end at maximum or minimum. They have two inflection points, near which it grows linearly. Growth rate is zero in middle. Double sigmoid curves change exponentially at beginning and end and near middle. Amount or percentage N over variable x is N(x) = (x - d) * (1 - exp(-((x - d) / s)^2)), where d is average, and s is standard deviation.

Verhulst curve

Logistic curves {Verhulst curve} can have growth rate directly related to current percentage or total amount and directly related to current resource amount. dN / dt = r * N * (1 - N/K), where N is total population, r is growth rate, and K is maximum population possible {carrying capacity, logistic}. Amount or percentage N over time t is N(t) = (K * N(0) * e^(r*t)) / (K + N(0) * (e^(r*t) - 1)), which comes from N(t) = A * (1 + m * e^(-t/T)) / (1 + n * e^(-t/T)), where A ~ 1, m ~ N(0), n ~ N(0) / K, and T = -1/r.

MATH>Geometry>Plane>Line

line

Lines {line} are straight and have no endpoint.

join

Line segments can connect {join} two points.

proportional division

Dividing line segment internally or externally can divide second line segment in same proportions {proportional division}. Put second-segment end on first-segment end. Then draw straight line between other endpoints. Then draw line parallel to straight line at first-line-segment dividing point.

rectilinear figure

Figures {rectilinear figure, line} can have only straight lines.

MATH>Geometry>Plane>Line>Axiom

Archimedes axiom for line

Line-segment multiples can have greater length than any line segment {Archimedes axiom, line} {axiom of Archimedes, line}.

Euclid twelfth axiom

Plane-geometry axioms {Euclid's twelfth axiom} {Euclid twelfth axiom} can claim that parallel lines exist.

parallel postulate

One and only one straight line through a point not on another straight line does not intersect second straight line {parallel postulate}|.

Playfair axiom

Euclid's twelfth axiom can be simpler {Playfair's axiom} {Playfair axiom}. Through any point, only one line can be parallel to fixed line. Given straight line and exterior point, only one straight line is parallel to the line.

MATH>Geometry>Plane>Line>Intersection

isometric axes

Three lines can meet at angles of 120 degrees {isometric axes}.

piercing point

Lines can intersect coordinate planes at one point {piercing point}, where they orthogonally project onto coordinate planes. Surfaces intersect coordinate planes to make curves {trace curve}.

quartic symmetry

Four symmetry axes {quartic symmetry} can make eight angles of 45 degrees.

transversal

A line {transversal} can cross two coplanar lines, to form four angle pairs. Angles can orient in same direction {corresponding angle, transversal}. Angles {alternate angle} can lie on opposite sides of coplanar-line transversal. Angles can be opposite each other at intersection points {vertically opposite angle}.

MATH>Geometry>Plane>Line>Theorem

intercept theorem

If two transverse lines cross parallel lines, transversal line-segment ratios are equal {intercept theorem}.

Pappus theorems

Pappus invented line and centroid theorems {Pappus's theorems} {Pappus theorems}.

Draw line segments from first first-line-segment point to first second-line-segment point, second first-line-segment point to second second-line-segment point, and first first-line-segment point to second second-line-segment point. The drawn line segments have three intersection points that make a straight line.

Arcs have planes. Arcs rotated around plane axis generate area equal to arc length times circular-path segment traveled by arc centroid.

Planes rotated around plane axis generate volume equal to surface area times circular-path segment traveled by centroid.

revolution

If centroid paths are circles, the previous two theorems generate surfaces or volumes of revolution.

MATH>Geometry>Plane>Line>Kinds

anti-parallel line

Two straight lines {anti-parallel line} can cut two straight lines so first angle cut by one line is supplementary to second angle cut by other line.

extended line

Line segments {extended line} can be longer, at either end.

isotopic line

Curved lines {isotopic line} can be perpendicular to themselves.

median of trapezoid

A line {median, trapezoid} joins midpoints of non-parallel trapezium or trapezoid sides.

nodal line

Figure lines {nodal line} can stay fixed during rotation or deformation.

oblique line

Lines {oblique line} can be neither parallel nor perpendicular to a direction.

MATH>Geometry>Plane>Polygon

polygon

Many-sided figures {polygon} can surround one connected space. Polygons {convex polygon} can have all interior angles less than 180 degrees. Polygons {concave polygon} can have at least one angle more than 180 degrees. Polygons {circumscribed circle} can have all vertices on a circle. Polygons {circumscribed polygon} can have all sides tangent to a closed curve.

apothem

Line segments {apothem} {shorter radius} can run from regular-polygon center perpendicular to side.

Brianchon theorem

Three lines, joining opposite vertices of a conic circumscribed hexagon, pass through one point {Brianchon's theorem} {Brianchon theorem}.

opposite side

Regular polygons can have parallel equal-length sides {opposite side, polygon}. Regular polygons can have opposite equal angles {opposite angle}. Regular polygons can have opposite vertices {opposite vertex}.

nodal point of polygon

A number {order, polygon} of polygons can share a point {nodal point, polygon}.

MATH>Geometry>Plane>Polygon>Number of Sides

ternary

Figures can have three parts {ternary}.

quaternary polygon

Figures can have four parts {quaternary}.

quinary

Figures can have five parts {quinary}.

senary

Figures can have six parts {senary}.

septenary

Figures can have seven parts {septenary}.

octenary

Figures can have eight parts {octenary}.

undenary

Figures can have eleven parts {undenary}.

MATH>Geometry>Plane>Polygon>Kinds

gnomon

Removing a smaller parallelogram, which shares parts of two adjacent sides, from a larger parallelogram makes a figure {gnomon}|.

pentagram of Pythagoras

The five regular-pentagon diagonals make a five-point star {pentagram of Pythagoras}|.

polyomino

Rectilinear plane figures {polyomino} can involve congruent squares that share sides. Finite numbers of identical squares can join at edges to make shapes, such as crosses or lines. One polyomino can tile plane periodically or not. One polyomino cannot tile plane aperiodically. Polyomino pairs or triples can tile plane periodically, aperiodically, or not. Because aperiodic tilings are possible, no algorithm can decide, for all sets, if a polygon set will tile plane.

regular polygon

Polygons {regular polygon} can have all angles equal and all sides equal.

tangram

Games {tangram} can use squares cut into seven pieces, which rearrange without overlapping to make designs.

MATH>Geometry>Plane>Polygon>Kinds>Number

trigon

Polygons {trigon} can have three sides.

tetragon

Polygons {tetragon} can have four sides.

pentagon polygon

Figures {pentagon}| can have five sides.

hexagon

Figures {hexagon}| can have six sides.

heptagon

Figures {heptagon}| can have seven sides.

octagon

Figures {octagon}| can have eight sides.

dodecagon

Figures {dodecagon}| can have 12 sides.

icosagon

Figures {icosagon}| can have 20 sides.

n-gon

Polygons {n-gon} can have n sides. In polygons, exterior-angle sum equals 360 degrees. In polygons, interior-angle sum is (n - 2) * (180 degrees).

MATH>Geometry>Plane>Polygon>Kinds>Quadrilateral

quadrilateral

Figures {quadrilateral}| can have four sides. Quadrilaterals {cyclic quadrilateral} can have all four vertices on a circle.

area: parallelogram

Parallelogram area = b * a * sin(A), where a and b are side lengths and A is angle between them.

area: rectangle

Rectangle area = l*h, where l and h are side lengths.

area: rhombus

Rhombus area = s * s * sin(A), where s is side length and A is small angle.

area: square

Square area = s^2, where s is side length.

area: trapezoid

Trapezoid area = a * sin(A) * (b1 + b2) / 2, where a is vertical-side length, A is small angle between side and base, and b1 and b2 are bases. Trapezoid area = h * (b1 + b2) / 2, where h is height and b1 and b2 are bases.

perimeter: rectangle

Rectangle perimeter = 2*a + 2*b, where a and b are side lengths.

perimeter: rhombus

Rhombus perimeter = 4*s, where s is side length.

perimeter: square

Square perimeter = 4*s, where s is side length.

perimeter: trapezoid

Trapezoid perimeter = a + b + c + d, where a, b, c, d are side lengths.

perimeter: parallelogram

Parallelogram perimeter = 2*a + 2*b, where a and b are side lengths.

quadrangle

Figures {quadrangle}| can have four vertices. Quadrangles {simple quadrangle} can have four vertices and four lines, with no diagonals. Quadrangles {complete quadrangle} can have four points, four lines, and two diagonals.

kite

Diamond-shaped quadrilaterals {kite} can have two equal-side pairs and two equal-angle pairs. Diagonals are perpendicular.

parallelogram

Figures {parallelogram}| can have two pairs of equal, opposite, and parallel sides.

rectangle

Figures {rectangle} can have two pairs of equal and opposite sides at right angles.

rhombus

Figures {rhombus}| {rhom} can have four equal sides.

square figure

Figures {square figure} can have four equal sides at right angles.

trapezoid

Figures {trapezoid}| {trapezium} can have only one pair of parallel opposite sides.

skew quadrilateral

Quadrilaterals {skew quadrilateral} can have four points not in same plane.

MATH>Geometry>Plane>Polygon>Kinds>Triangle

triangle

Plane figures {triangle} can have three sides.

area

Triangle area equals 0.5 * b * h, where b is base and h is height.

Triangle area = r*s, where r is inscribed-circle radius, s is (a + b + c) / 2, and a, b, c are sides.

Triangle area = c^2 * sin(A) * sin(B) / (2 * sin(C)), where c is side length, and A, B, C are opposite angles to sides a, b, c.

Triangle area = 0.5 * b * c * sin(A), where b is base length, c is side length, and A is angle between base and side.

area: isosceles

Isosceles-triangle area = 0.5 * b * a * sin(A), where b is base length, a is equal-side length, and A is base angle.

area: equilateral

Equilateral-triangle area = 3^(0.5) * s / 2, where s is side length.

angle sum

Triangle angle sum is 180 degrees.

triangle perimeter

Triangle perimeter = a + b + c, where a, b, c are side lengths. Isosceles-triangle perimeter = 2*a + b, where a is equal-side length, and b is other-side length. Equilateral triangle perimeter = 3*s, where s is side length.

congruent

Triangles {congruent}| can be the same but have different locations. Congruent triangles have same three sides, same two angles with same side, and same two sides with same angle.

Heronic triple

Three integers {Heronic triple} can represent triangle sides for triangles with integer area.

Hero formula

Triangle area = (s * (s - a) * (s - b) * (s - c))^0.5, where s = 0.5 * (a + b + c) and a, b, c are sides {Hero's formula} {Hero formula}.

nine-point circle

For triangles, a circle {nine-point circle} can pass through side midpoints, feet of perpendiculars to sides, and midpoints of line segments between orthocenter and triangle vertices. Nine-point circle center is equidistant to orthocenter and circumcenter.

Pythagorean theorem

Right triangles have one right angle. In Euclidean geometry, for right triangles, sum of squares of two shorter sides equals hypotenuse squared {Pythagorean theorem}: c^2 = a^2 + b^2.

proof

To prove theorem, use geometric construction. Use only straightedge and compass to draw new lines and angles. See Figure 1.

Square sides. See Figure 2.

Add original triangle of size 0.5 * a * b, triangle of size 0.5 * a * b beside it, and rectangle of size a*b to squares of sides, to make square of sum of sides and complete the square: (a + b)^2. See Figure 3. (a + b)^2 = a^2 + b^2 + a*b + 0.5 * a * b + 0.5 * a * b = a^2 + b^2 + 2*a*b.

Flip hypotenuse square into square of sum of sides. See Figure 4. c^2 + 4 * (0.5 * a * b) = (a + b)^2. c^2 + 2*a*b = a^ + 2*a*b + b^2. c^2 = a^ + b^2. Hypotenuse squared equals sum of squares of two shorter sides.

Figure 1

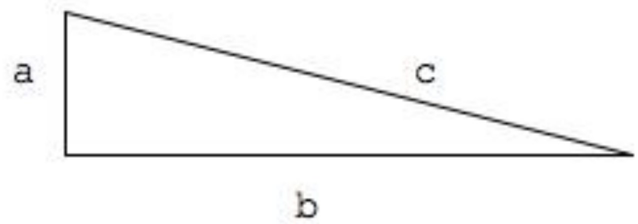


Figure 2

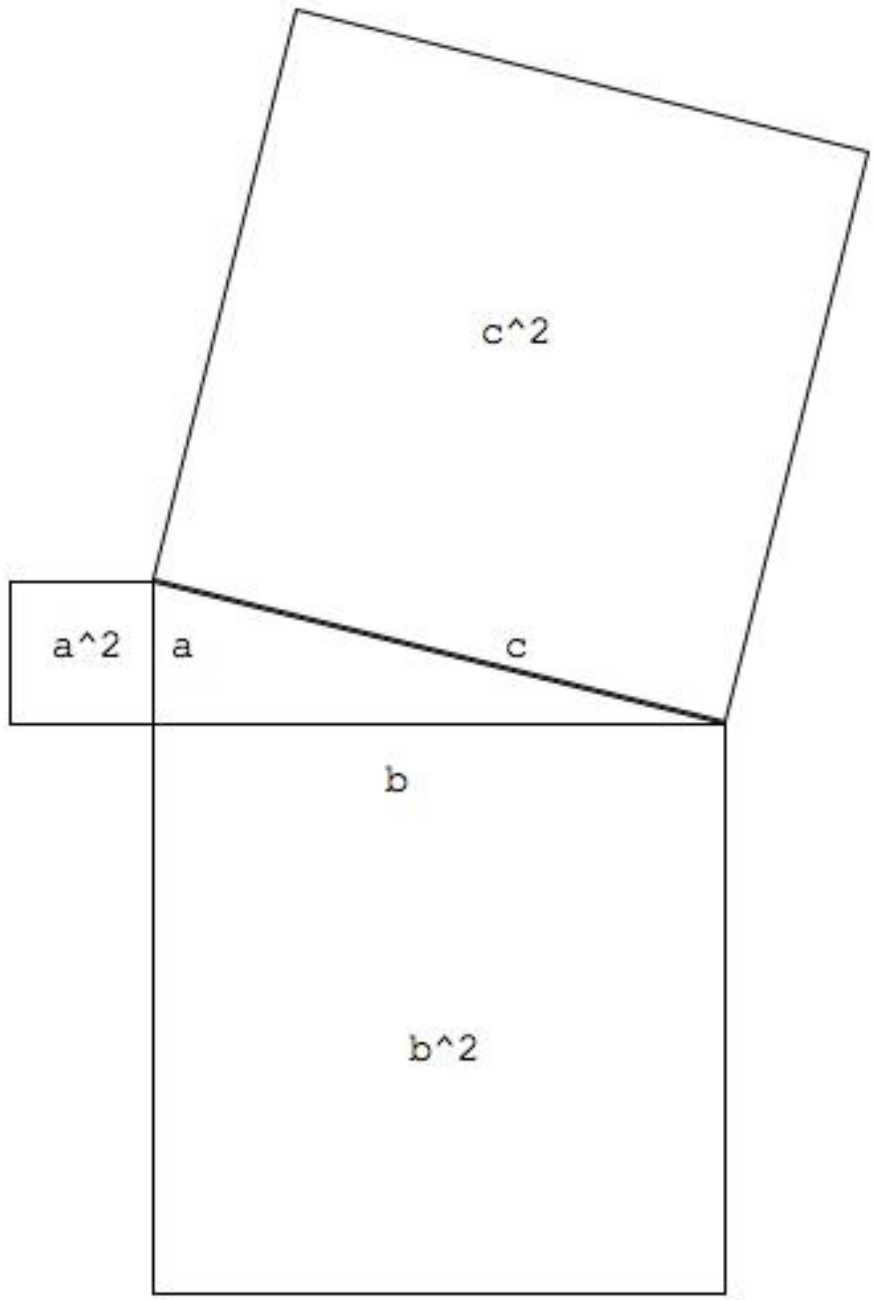


Figure 3

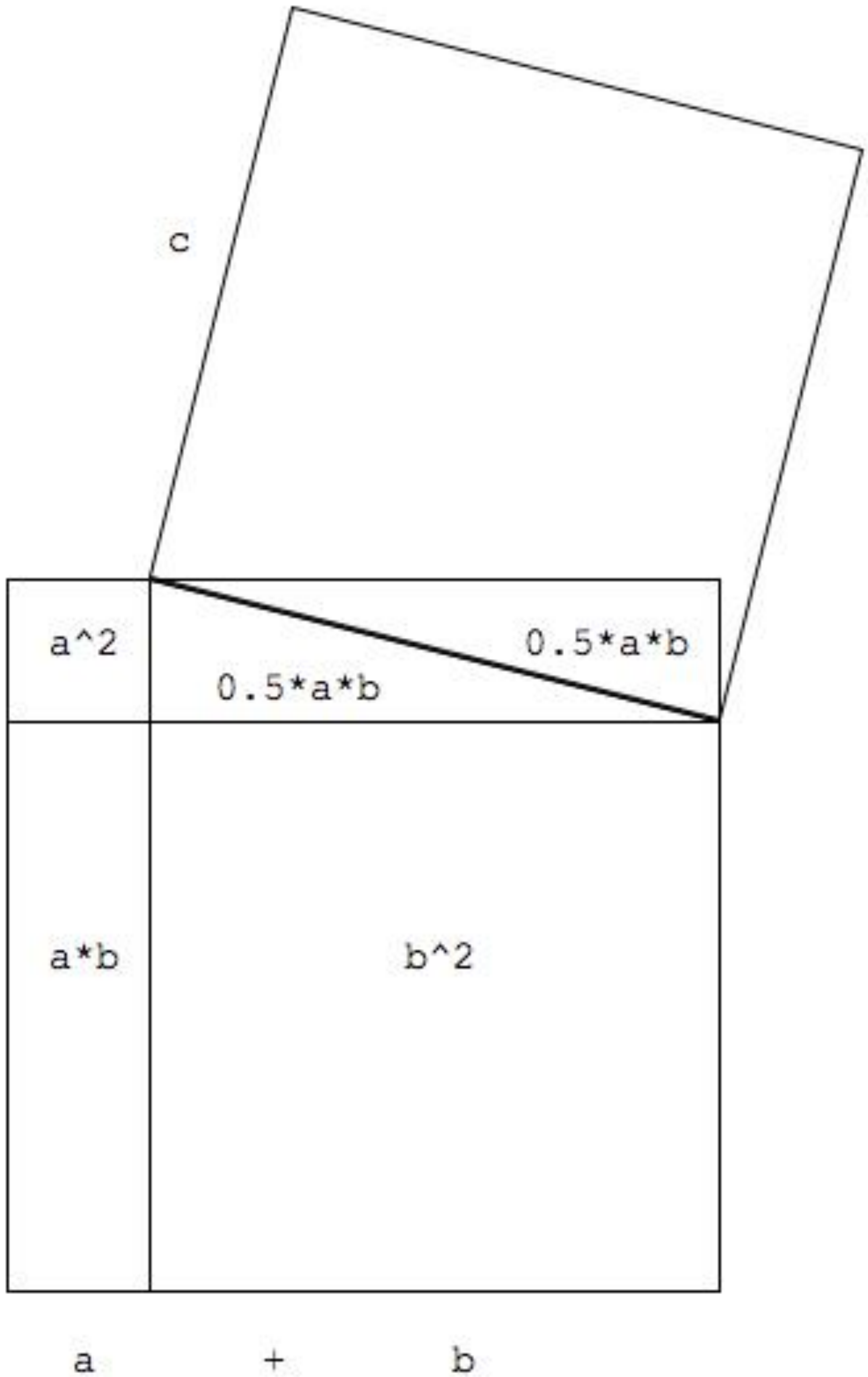
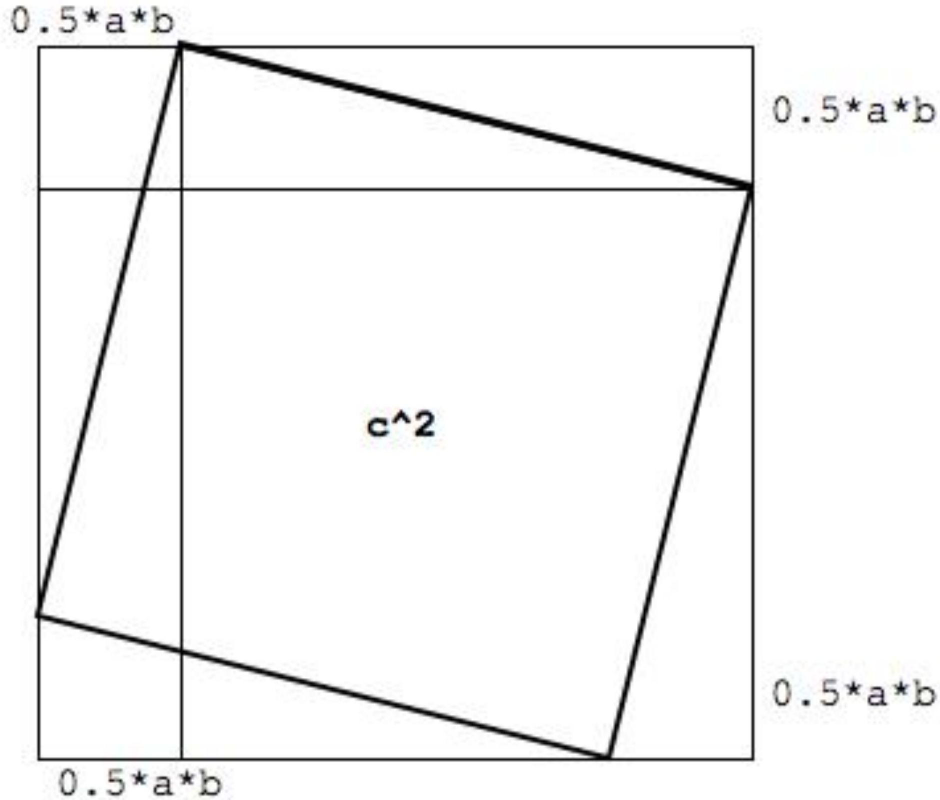


Figure 4



triangle inequality

For three points, distance between first two points is less than or equal to sum of distance between first and third point and distance between second and third point {triangle inequality}|.

triangulation length

To find side length, first measure base line, then measure angles to other point, and then compute side length {triangulation, length}|. To find angle, first measure base line, then measure sides, and then compute angle {chain triangulation}.

trilateration

To find space position, first measure distance to three reference points, then find intersection of three spheres {trilateration}|. Global Positioning System (GPS) uses 24 fixed satellites and trilateration by timing signals.

MATH>Geometry>Plane>Polygon>Kinds>Triangle>Line

altitude of triangle

Triangles have line segment {altitude}| from vertex perpendicular to opposite side.

arm and leg

Right triangles have two shorter sides {arm, triangle} {leg, triangle}.

base of triangle

Triangles have a side {base, triangle} intersected by the altitude.

hypotenuse

Right triangles have a longest side {hypotenuse, triangle}|.

median of triangle

Triangles have line segments {median, triangle} from vertices to opposite-side midpoints.

MATH>Geometry>Plane>Polygon>Kinds>Triangle>Point

Brocard point

Inside triangles, lines from vertexes can meet at two points {Brocard point} and form equal angles at intersections with sides.

circumcenter

Triangle circumscribed circles have centers {circumcenter} inside triangle.

median point

Three medians intersect at one point {median point}.

orthocenter

Triangles have a point {orthocenter} where three altitudes intersect.

MATH>Geometry>Plane>Polygon>Kinds>Triangle>Kinds

acute triangle

Triangles {acute triangle} can have largest angle less than 90 degrees.

equiangular triangle

Triangles {equiangular triangle} can have all angles equal 60 degrees.

equilateral triangle

Triangles {equilateral triangle} can have all sides equal.

isosceles triangle

Triangles {isosceles triangle}| can have two sides equal.

obtuse triangle

Triangles {obtuse triangle} can have largest angle more than 90 degrees.

pedal triangle

From fixed point, lines to vertexes can be perpendiculars {pedal triangle}. Pedal triangles are lines {pedal line} {Simpson's line} if fixed point is on circumscribed circle.

Pythagorean triangle

Right triangles {Pythagorean triangle} can have integer-length sides, such as 3, 4, and 5 {rope stretcher's triangle, Pythagorean triangle}; 5, 12, and 13; or 8, 15, and 17.

right triangle

Triangles {right triangle}| can have one angle of 90 degrees.

rope stretcher's triangle

Right triangles {rope stretcher's triangle} can have side lengths 3, 4, and 5.

scalene triangle

Triangles {scalene triangle} can have no two sides equal.

similar triangle

Triangles {similar triangle} can have same ratios of sides. Similar triangles have corresponding sides and angles.

spherical triangle

Triangles {spherical triangle} on spheres can have three right angles {trirectangular spherical triangle} or two right angles {birectangular spherical triangle}.

MATH>Geometry>Plane>Tiling

tiling

Different-shape polygons can cover planes {tiling, geometry} with no gaps and no overlaps.

shapes

One triangle, square, or hexagon shape can tile. One pentagon shape cannot tile.

Pairs of shapes can tile, such as two different irregular pentagons. Such tilings have repeated parallelograms {periodic tiling}.

Spirals can tile without repeated parallelogram {aperiodic tiling}. For example, nine-sided triangle-shaped tile {versatile} can tile periodically and non-periodically. Square-shaped tiles with protruding points and corresponding concave depressions can tile aperiodically. Other four-sided shapes {Penrose tiles} have protrusions and depressions, have five-fold symmetry, make repeated patterns, and can tile aperiodically {quasi-periodic tiling}.

periodicity

Algorithms can decide if tiles can tile the plane periodically. No algorithm can decide generally if tiles can tile the plane aperiodically.

tessellation

Identical shapes, such as triangles, rectangles, hexagons, or special five-sided polygons, or shape sets can fill planes, polyhedrons, or curved surfaces without gaps or overlaps {tessellation}|.

types

Tessellation {regular tessellation} can use equilateral triangles, squares, or regular hexagons. Tessellation {homogeneous tessellation} {semiregular tessellation} can have congruent common vertices {nodal point, homogeneous tessellation} and regular polygons. Tessellation {non-homogeneous tessellation} can use irregular shapes, different sizes of one shape, or both.

MATH>Geometry>Projections

projection

Projection {projection, geometry} can project object onto plane or planes or project sphere onto plane.

perspective projection

Pictures of scenes can use linear perspective {perspective projection}.

MATH>Geometry>Projections>Object Onto Planes

geometric projection

Objects can project {geometric projection} onto planes. Plane figures {plane of projection} {projection plane} {image-plane} can transform to objects {image-picture} by one-to-one correspondence. Lines {projection ray} can join corresponding points from figures to images.

axonometric projection

Projections {axonometric projection} can keep vertical lines vertical, have main horizontal axis at angle 45 or 30 degrees to verticals, and have third dimension at angle of 45 or 60 degrees to main horizontal axis.

isometric projection

Projections {isometric projection} can have all vertical lines stay vertical, and both other axes at angle 60 degrees to verticals.

oblique projection of object

Projections {oblique projection, object} can keep all vertical lines vertical, have main horizontal axis at right angle to right of verticals, and have third axis at angle 45 degrees to left of verticals. Oblique projection has projection rays not perpendicular to the plane.

MATH>Geometry>Projections>Object Onto Planes>Orthogonal

orthogonal projection

Parallel projection {orthogonal projection, geometry} {orthographic projection, orthogonal} can have projection rays perpendicular to the plane. Orthogonal projection can be on horizontal plane {horizontal projection} {plan, projection}, vertical plane {elevation, projection}, or other planes {section, projection}. Plans {figured plan} can mark vertical distances from plane to figure.

first angle projection

Orthogonal projection {first angle projection} can be on bottom horizontal plane {plan projection, first angle}, back left vertical plane {front elevation, first angle}, and back right vertical plane {side elevation, first angle} {end elevation, first angle}. Front and plan are like looking into far lower corner from outside room.

third angle projection

Orthogonal projection {third angle projection} can be on top horizontal plane {plan projection, third angle}, front right vertical plane {front elevation, third angle}, and front left vertical plane {side elevation, third angle} {end elevation, third angle}. Front and plan projections are like looking into top front corner from outside room. This projection makes common edges closer and is usually better.

MATH>Geometry>Projections>Object Onto Planes>Pictorial

pictorial projection

All three dimensions can be in one picture {pictorial projection}.

metric projection

Pictorial projection {metric projection} can be to scale.

MATH>Geometry>Projections>Rays

parallel projection

Projection rays can be parallel {cylindrical projection, rays} {parallel projection, rays}, so projection center is at infinity. Parallel projection can have projection rays perpendicular to plane {orthogonal projection, parallel} or not perpendicular to plane {oblique projection, parallel}.

radial projection

Projection rays can pass through fixed point {central projection, rays} {radial projection} {conical projection, rays}.

MATH>Geometry>Projections>Descriptive Geometry

descriptive geometry

Three-dimensional object can project onto two orthogonal planes {descriptive geometry} in central projection, point projection, parallel projection, perspectivity, or projectivity.

central projection of object

Three-dimensional object can project onto two orthogonal planes from center {central projection, geometry}.

parallel projection descriptive

Three-dimensional object can project onto two orthogonal planes using parallel lines {parallel projection, descriptive geometry}.

perspectivity

Three-dimensional object can project onto two orthogonal planes using section through object and projection onto plane {perspectivity}.

point projection

Three-dimensional object can project onto two orthogonal planes from point {point projection}.

projectivity

Three-dimensional objects can project onto two orthogonal planes using projection and section sequences {projectivity}.

MATH>Geometry>Projections>Sphere To Plane

conical projection

Projecting sphere onto plane {conical projection, geometry} can have meridians radiating from vertex at equal angles and parallels in concentric circles around vertex.

equatorial projection

Equator can be in map center {equatorial projection}. Equatorial projection can have two forms {semisided} {flat polar quartic}.

line projection

Projecting sphere onto plane {line projection} can keep east-west horizontal latitude constant and vertical longitude lines from pole to pole constant.

orthomorphic projection

Projecting sphere onto plane {orthomorphic projection} can keep shapes similar.

MATH>Geometry>Projections>Sphere To Plane>Area Constant

equal area projection

Projecting sphere onto plane {homalographic projection} {equal area projection} can keep area ratios constant.

cylindrical projection

Projecting sphere onto plane {cylindrical projection, geometry} can make equator horizontal, make meridians perpendicular to equator and equidistant from each other, and place parallels at distances that maximally reduce distortion. Cylindrical projections preserve areas.

elliptical projection

Projecting sphere onto plane {elliptical projection} can place equator, meridians, and parallels to maximally reduce distortion. Elliptical projections preserve areas.

orthographic projection

Projecting sphere onto plane {orthographic projection, area} can use parallel rays to place one hemisphere on its equatorial plane, making circle edges highly distorted. Orthographic projection preserves either equivalent angles or areas. Mapmaker can choose to keep equivalent areas or equivalent directions but cannot choose both.

MATH>Geometry>Projections>Sphere To Plane>Azimuth Constant

azimuthal projection

Projecting sphere onto plane {azimuthal projection} can keep correct azimuth.

conformal transformation

Projecting sphere onto plane {conformal transformation}, such as stereographic, Mercator, and Miller projections, can preserve directions and angles. Gnomic, cylindrical, and elliptical projections do not preserve directions and angles.

Mercator projection

Projecting sphere onto plane {Mercator's projection} {Mercator projection} can maintain straight-line loxodrome constant bearings.

Miller projection

Projecting sphere onto plane {Miller projection} can keep straight-line loxodrome constant bearings but reduce area distortion at poles.

MATH>Geometry>Projections>Sphere To Plane>Zenithal

zenithal projection

Projecting sphere onto plane {zenithal projection} can use plane tangent to sphere.

MATH>Geometry>Projections>Sphere To Plane>Zenithal>Center

central projection of sphere

Projecting sphere onto plane {central projection, mapping} {gnomic projection} can use zenithal projection with projection center at sphere center.

orthographic projection zenithal

Projecting sphere onto plane {orthographic projection, zenithal} can use zenithal projection with projection center at infinite distance. Orthographic projection preserves either equivalent angles or areas. Mapmaker can choose to keep equivalent areas or equivalent directions but cannot choose both.

stereographic projection

Projecting sphere onto plane {stereographic projection, map} can use zenithal projection with projection center at opposite end of diameter at tangent.

MATH>Geometry>Projections>Sphere To Plane>Zenithal>Tangent

normal projection

Projecting sphere onto plane {normal projection} can use plane tangent at equator.

oblique projection zenithal

Projecting sphere onto plane {oblique projection, zenithal} can use plane tangent to sphere at point that is neither at pole nor on equator. Pole can be not at map center.

polar projection

Projecting sphere onto plane {polar projection} can use plane tangent at pole. Pole can be at map center {transverse projection} or offset from center.

MATH>Geometry>Solid

solid geometry

Geometry {solid geometry} can be about three spatial dimensions and solids.

solid figure

Closed surface can bound figure {solid figure}. Solid can be figure bounded by planes {face, solid}, which meet at points {vertex, solid} and lines {edge, solid}.

parallelepiped

Six-sided solids {parallelepiped}| can have six parallelogram faces.

pencil of figures

Figure sets {pencil, geometry} can have all figures pass through common points. Lines can pass through one point {vertex, pencil}. Circles can pass through two points. Order-n curves can pass through n^2 points. Parallel figures can have a common point {ideal point} or line {ideal line}. Parallel planes can have a common line {axis, plane}. Spheres can have a common circle. Planes can share a straight line.

torus

Rectangles can transform {torus}|, by connecting top and bottom to make a cylinder and left and right to make a circle. Toruses have two circles, one along cylinder and one around cylinder. Toruses can result when closed plane regions revolve around an axis in a plane that does not intersect surface. If closed plane regions are circles, solid rings form, with volume = 2 * pi^2 * p^2 * r^2 and area = 4 * pi^2 * p * r, where p is torus radius and r is circle radius.

volume of solid

Solid figure has interior space {volume, solid}. Box volume = h*w*l, where h, w, l are side lengths.

MATH>Geometry>Solid>Angle

solid angle

Two solids meet at an angle {solid angle}| measured as a wedge. Solid angle measure is in steradians. Solid angle is at vertex.

dihedral angle

Two angles {dihedral angle}| are where two planes intersect. The smaller angle is angle size {inclination}. Dihedral angle lies between line in each plane perpendicular to intersection line {vertex, dihedral angle}. Dihedral angle is angle between normals to the planes. Dihedral angle measure is in radians.

plane angle

Two planes meet at an angle {plane angle} measured between two lines perpendicular to plane-intersection line. Plane angle measure is in radians.

polyhedral angle

Edges and faces at vertices make solid angles {polyhedral angle}|.

trihedral angle

Three lines can meet at one point, defining three planes {trihedral angle}|. Trihedral-angle ray relative directions can be left-handed or right-handed.

MATH>Geometry>Solid>Cone

cone in geometry

Angles can rotate through 360 degrees, around the line {bisector, cone} dividing angle in half, to make solid figure {cone, geometry}. Cones have base, bisector axis, vertex, and vertex angle. Cone lines {element, cone} can go through vertex. Right-circular-cone area is bottom area plus side area: pi * r^2 + pi * r * (r^2 + h^2)^0.5, where r is base radius and h is height. Right-circular-cone volume = (pi * h * r^2) / 3, where r is radius and h is height.

nappe

Conical surfaces have two conical halves {nappe}|.

slant height of cone

Right circular cones have generating-line length {slant height, cone}.

MATH>Geometry>Solid>Cone>Conic Section

conic section

Planes can intersect cones to make plane curves {conic section}|. Plane can be parallel to base and intersect cone at right angle to axis {circle, cone}. Plane can intersect cone at angle less than vertex angle {ellipse, cone}. Plane can intersect cone at angle equal to vertex angle and parallel to element {parabola, cone}. Plane can intersect cone parallel to axis {hyperbola, cone}. Plane can intersect cone so plane includes vertex and bisector {intersecting lines}. Plane can be tangent to cone {tangent line, cone}.

slope

Circles and ellipses are closed curves and have same slope at diameter ends. Parabolas are not closed curves and approach maximum slope as they go farther from axis. Hyperbolas are not closed curves and approach maximum slope as they go farther from axis.

pole

Two tangents to conic can meet at point {pole, cone}.

conic points

Two conics intersect at four points. Two real conics that do not intersect share two imaginary chords.

generation: point and curve

For conic sections, line goes through fixed point {generator, cone} and closed curve {directrix, cone}.

generation: lines

Conics can be line series and pencils, as can ruled quadrics.

truncate

Two non-parallel planes can cut {truncate} cone, cylinder, prism, or pyramid.

MATH>Geometry>Solid>Cone>Conic Section>Sections

ellipse

(x - h)^2 / a^2 + (y - k)^2 / b^2 = 1, where center is at (h,k), a is longer radius, and b is shorter radius {ellipse, conic}|. x = - h + a^2 / (a^2 + b^2)^0.5, where a > b.

foci

Ellipses have two focuses. Ellipse points have distances to foci. For all ellipse points, distance sum is constant.

Ellipses are symmetric about two lines. Ellipses have four points {vertex, ellipse} intersected by symmetry axes. Longest symmetry axis {major diameter} {major axis} has length = 2*a, where a > b. Shortest symmetry axis {minor diameter} {minor axis} has length = 2*b.

circle

Circle equation is (x - h)^2 + (y - k)^2 = r^2, where r is radius, and center is at (h,k).

auxiliary circle

A circle {auxiliary circle} with diameter equal major axis can surround ellipse.

helix

Curves {helix}| {bolt, helix} can maintain constant angle with cylinder, cone, or sphere generator.

In right circular cylinders, helix {circular helix} has equations x = r * cos(A), y = r * sin(A), z = r * A * cos(B), where A is revolution angle, r is cylinder radius, and B is helix-to-generator inclination angle.

Right-circular-cone helices are like tapered screws. If helices open to become circle sectors, they look like equiangular-spiral pieces.

spherical helix

Loxodromic spirals can be helices {spherical helix}.

hyperbola

In Cartesian coordinates, hyperbolas {hyperbola, conic}| have equation (x - h)^2 / a^2 - (y - k)^2 / b^2 = 1, where center is (h,k), a is length between vertex and center, and b is length between focus and hyperbola point along a line perpendicular to long axis through focus.

Eccentricity e is distance from hyperbola point to focus divided by distance from hyperbola point to directrix and is constant and greater than 1: e = (a^2 + b^2)^0.5 / a. If center is at (0,0), focus is at x = a * e.

In polar coordinates with center at origin, r^2 = a^2 * b^2 / (b^2 * cos^2(A) - a^2 * sin^2(A)). In polar coordinates with center at focus, equation applies to only one branch: r = a * (e^2 - 1) / (1 - e * cos(A)) = a * ((a^2 + b^2)/a^2) / (1 - ((a^2 + b^2)^0.5 / a) * cos(A)), where -1 <= cos(A) <= 1.

directrix

Hyperbolas have two directrixes, a fixed line perpendicular to the long axis, typically between center and vertex, in the same plane as the hyperbola.

foci

Hyperbolas have two focuses, a fixed point on the long axis on the convex side. Hyperbola points have distances to foci. All hyperbola points have the same focal-distance difference, equal to 2*a.

symmetry

Hyperbolas are symmetric about the centers. The symmetry line intersects hyperbola at two points {vertex, hyperbola}.

Hyperbolas can rotate around long axis to make hyperboloid surfaces.

diameters

A line segment {transverse diameter} between vertices has length 2*a. A line segment {conjugate diameter} perpendicular to transverse diameter at focus has length 2*b. Hyperbolas {equilateral hyperbola} can have transverse diameter equal to conjugate diameter. The auxiliary circle, with center at (0,0) and radius a, intersects the vertices.

asymptote

When x is large positive or negative, hyperbola slope approaches straight line {asymptote, hyperbola}.

rectangular hyperbola

If transverse and conjugate axes are equal, hyperbola {rectangular hyperbola} can have asymptotes at right angles. If rectangular hyperbola is symmetric to coordinate axes, equation is x^2 - y^2 = a^2, where a is half axis length. If asymptotes are coordinate axes, equation is x*y = a^2 / 2 = c^2, where a is half axis length and c is constant.

auxiliary rectangle

Conjugate diameter determines rectangle {auxiliary rectangle} between the hyperbola curves.

parabola as conic section

Conic sections {parabola, conic section} can have U shape.

equation

Parabola equation can be a * (x - h) = (y - k)^2, where h is x-intercept, k is y-intercept, and a is major conic-section diameter. Minor conic-section diameter is zero. For parabola, x = k - a. Parabola equation can be y = a*x^2 + b*x + c.

definition

Distance from any parabola point to parabola center {focus, parabola} equals distance from point to defining line {directrix, parabola}.

axis

A symmetry line {axis, parabola} divides parabolas lengthwise. Axis intersects parabola at point {vertex, parabola}. Distance {focal length, parabola} from focus to vertex is major diameter. Parabolas have no minor diameter.

semicubical parabola

Equation a * y^2 = x^3 or b * y^3 = x^2, where a is focal length, defines parabola {semicubical parabola}.

MATH>Geometry>Solid>Cone>Conic Section>Eccentricity

eccentricity of conic

Distance from conic-section point to focus divided by distance from conic-section point to directrix is constant {eccentricity, conic}: e = (a^2 + b^2)^0.5 / a, where a is major axis and b is minor axis. For circle, e = 2^0.5. For parabola, e = 1, because b = 0. For hyperbola and ellipse, e > 1.

eccentric angle

For ellipses, angle A {eccentric angle} in equations x = a * cos(A) and y = b * sin(A), where a is ellipse major axis, and b is ellipse minor axis, determines eccentricity. Hyperbola has eccentric angle A with x = a * sec(A) and y = b * tan(A).

eccentric circle

In ellipses, two circles {eccentric circle} can have major and minor axes as diameters. In hyperbola, two eccentric circles have transverse axis and conjugate axis {harmonic conjugate of transverse axis} as diameters.

polar equation of conics

l / r = 1 - e * cos(A) {polar equation}, where l is latus-rectum length divided by two, r is distance from pole or focus, e is eccentricity, and A is polar angle. Transverse or major-axis positive direction is reference line {initial line} {polar axis, conic}.

MATH>Geometry>Solid>Cone>Conic Section>Parameters

directrix

For conic sections, a line {directrix, conic} goes through generator point and closed curve.

focal length

Distance {focal length, conic} from focus to vertex is major diameter.

latus rectum

Line segments {latus rectum} can go through conic-section focus and two conic-section points, whose distances to focus are equal.

polar of conic

Line segments {polar, conic} can connect conic poles.

MATH>Geometry>Solid>Cube

cube

Six-sided solids {cube} can have six square faces. Cube volume = s^3, where s is side. Skeletons of n-dimensional cubes correspond to Gray binary codes.

cuboid

Six-sided solids {cuboid} can have six rectangular faces.

MATH>Geometry>Solid>Curve

curve of circles

Lines {curve, circles} can vary curvature radius. Curves {regular arc} {regular curve} can be in intervals.

arc length

Surfaces have metric elements {arc length, curve}: ds^2 = dx^2 + dy^2 + dz^2. Locally, arc length is invariant. Arc-length curvature and functions {torsion, curve} can determine space-curve properties, except position.

Plucker formula

Curve order and class relate to simple singularities {Plucker formula}.

Gauss characteristic equation

Curvature depends only on surface parameters {Gauss characteristic equation}. Curvature is product of principal curvatures. Linear curvature is tangent-angle change divided by arc length. It is radial-length change times two divided by arc length squared. It is curved-surface solid angle divided by surface area. To measure curvature around point, use regular hexagon and measure angles.

MATH>Geometry>Solid>Curve>Kinds

binormal
All surface points have a perpendicular {binormal} to osculating plane.

coaxial circle
In three dimensions, circles {coaxial circle} can share axis.

higher curve
Curves {higher curve} {higher plane curve} can have equations with degree greater than two. Defining degree-n curves requires n * (n + 3) / 2 points. Higher plane curves have inflection points, multiple points, cusps, conjugate points, genuses, and branches. One degree-m curve and one degree-n curve can intersect at m*n points.

osculating curve
A point and two points on a curve near the point define a circle with curvature equal to curve {osculating curve}| curvature.

screw curve
Rotation about, and translation along, line makes space curve {screw curve}.

simple closed curve
Non-intersecting curves {simple closed curve} can enclose regions. Simple closed curves {simply connected region} can surround only region points.

skew curve
Curves {skew curve} {twisted curve} can go outside of plane.

MATH>Geometry>Solid>Cylinder

cylinder
Solid figures {cylinder} can have circle and long axis. Right-circular-cylinder area equals top area plus bottom area plus side area = 2 * pi * r^2 + 2 * pi * r * h, where r is base radius and h is height. Right-circular-cylinder volume = pi * h * r^2, where r is radius and h is height.

MATH>Geometry>Solid>Ellipsoid

ellipsoid
Surfaces {ellipsoid}| can have elliptical cross-sections in three coordinate planes. Larger axes can be equal {oblate ellipsoid}. Smaller axes can be equal {prolate ellipsoid}. For sphere, three axes are equal.

MATH>Geometry>Solid>Line

diagonal
Lines {diagonal} can go from one vertex to non-adjacent vertex. Tetrahedron and Csaszar polyhedron have no diagonals.

perpendicular lines
If two lines are perpendicular {perpendicular lines}, line slopes are negative reciprocals: m2 = -1 / m1, where m1 and m2 are slopes.

slant height of pyramid

Regular pyramid has apex and lateral face, which has median {slant height, pyramid} through apex.

skew line

Space can have non-intersecting, non-parallel lines {skew line}.

MATH>Geometry>Solid>Plane

plane of solid

Three points not on same line define a flat surface {plane, mathematics}. Lines and line points are in a plane. Lines perpendicular to all plane lines are perpendicular to plane. At a plane point, only one line can be perpendicular to a plane. At a line point, only one plane can be perpendicular to a line. If normal to a plane is perpendicular to a line, line and plane are parallel.

half-plane

Straight line divides plane in half {half-plane}.

multifoil

Equal congruent arcs can bound plane figure {multifoil}: three arcs {trefoil, figure}, four arcs {quatrefoil}, five arcs {pentafoil}, and six arcs {hexafoil}. Arc centers make regular-polygon vertices.

osculating plane

Planes {osculating plane} can pass through a surface point and two nearby points.

parabolic segment

Parabolas and chords, perpendicular to parabola axis, can make plane figures {parabolic segment}. Parabolic-segment area is 2 * c * a / 3, where c is chord length, and a is distance from vertex to chord.

radical plane

Eliminating spherical-equation second-power terms defines a plane {radical plane}. Radical plane contains circle of sphere pencil.

solid of revolution

Plane region rotated around line {axis of revolution} {revolution axis} in the plane makes solid {solid of revolution}. Plane-region perimeter generates surface {surface of revolution}. Volume is integral from x = a to x = b of pi * y^2 * dx, for y = f(x). Area is integral from x = a to x = b of 2 * pi * y * (1 + (dy/dx)^2)^0.5 * dx.

MATH>Geometry>Solid>Plane>Form

intercept form of plane

Plane can have equation x/a + y/b + z/c = 1 {intercept form, plane}, where a, b, c are x-axis, y-axis, and z-axis intercepts.

normal form of plane

Plane can have equation x * cos(a) + y * cos(b) + z * cos(c) = p {perpendicular form} {normal form, plane}, where p is perpendicular distance from origin to plane, and a, b, c are angles between perpendicular and x y z axes.

three-point form

Plane can have matrix |x y z 1 / x1 y1 z1 1 / x2 y2 z2 1 / x3 y3 z3 1| {three-point form}, where (xi,yi,zi) are points.

MATH>Geometry>Solid>Plane>Intersection

intersecting planes

Two planes are either parallel or intersecting {intersecting planes}. Intersecting planes make a wedge.

section

Plane and solid intersect to makes a plane region {section, plane} {cross-section}|.

sheaf of planes
Plane sets {sheaf, plane}| can pass through a point {center, sheaf}.

wedge of planes
Planes can intersect to make a solid figure {wedge, plane}|.

MATH>Geometry>Solid>Polyhedron

polyhedron in geometry
Simple multiple-sided solids {polyhedron, solid}| have genus zero.

edge
Faces can meet at lines {edge, polyhedron}.

face
Plane polygons {face, polyhedron} can bound solids.

vertex of polyhedron
Three or more edges can meet at points {vertex, polyhedron}.

MATH>Geometry>Solid>Polyhedron>Kinds

Csaszar polyhedron
Polyhedrons {Csaszar polyhedron} can model seven-color maps of toruses, finite projective planes, and error-correcting binary codes, when used as Hadamard matrices {Room square}.

Platonic solid
Polyhedrons {regular polyhedron} {regular solid} {Platonic solid} can have same regular polygon for all faces: four equilateral triangles {regular tetrahedron}, six squares {cube, Platonic solid}, six regular hexagons {regular hexahedron}, eight equilateral triangles {regular octahedron}, twelve regular pentagons {regular dodecahedron}, and twenty equilateral triangles {regular icosahedron}. All vertices are on circumscribed sphere. Concave regular polyhedrons are small stellated dodecahedron, great dodecahedron, or great icosahedron.

MATH>Geometry>Solid>Polyhedron>Kinds>Prism

prism
Congruent parallel faces {base, prism} and congruent parallelograms {lateral face}, joining corresponding base vertices, can make solids {prism}|. Lateral faces can be rectangles {right prism}. Prisms have adjacent lateral faces {prismatic surface}.

prismatoid
Polyhedrons {prismatoid} can have two faces that are parallel planes, with no vertices outside the faces. Lateral faces are triangles or quadrilaterals.

prismoid
Prisms {prismoid} can have quadrilaterals for all lateral faces. Bases have same number of sides and vertices.

pyramid
Prismatoids {pyramid} can have polygon bases, which contain all vertices except apex. Lateral faces are triangles. For tetrahedrons, bases are triangles. In regular pyramids, regular polygons can be bases and isosceles triangles can be lateral faces.

MATH>Geometry>Solid>Polyhedron>Kinds>Number Of Faces

tetrahedron

Polyhedrons {tetrahedron}| can have four faces.

pentahedron
Polyhedrons {pentahedron}| can have five faces.

diamond
Polyhedrons {diamond} can have six equal equilateral triangle faces. Diamonds are two tetrahedrons that share a face.

hexahedron
Polyhedrons {hexahedron} can have six faces.

rhombohedron
Polyhedrons can have six rhombus faces {rhombohedron}|.

octahedron
Polyhedrons {octahedron}| can have eight faces.

dodecahedron
Polyhedrons {dodecahedron}| can have 12 faces.

icosahedron
Polyhedrons {icosahedron}| can have 20 faces.

MATH>Geometry>Solid>Polyhedron>Kinds>Kepler-Poinsot

Kepler-Poinsot concave solid
Concave regular polyhedrons {Kepler-Poinsot concave solid} can be small stellated dodecahedron, great dodecahedron, or great icosahedron.

great dodecahedron
Concave regular polyhedrons can have 12 regular pentagons {great dodecahedron}.

great icosahedron
Concave regular polyhedrons can have 20 equilateral triangles {great icosahedron}.

small stellated dodecahedron
Concave regular polyhedrons can have 12 regular pentagons {small stellated dodecahedron}.

MATH>Geometry>Solid>Sphere

sphere
Solids {sphere} can result when semicircle rotates around its diameter. Equation is x^2 + y^2 + z^2 <= r^2, where r is radius.
area
Area is 4 * pi * r^2, where r is radius.
volume
Volume is 4 * pi * r^3 / 3, where r is radius.
imaginary circle
Two spheres share imaginary circle.
secondaries
Great circles can pass through poles.
spherical distance
Geodesic has length {spherical distance}.
spherical polygon
Great-circle arcs can bound spherical surface region {spherical polygon}.

spherical surface

$x^2 + y^2 + z^2 = r^2$ defines spherical surface. Area is 4 * pi * r^2, where r is radius.

diameter

Diameter is perpendicular to sphere at both endpoints.

coordinates

Sphere coordinates are longitude (360 degrees) and latitude (180 degrees), because they define unique points. Longitudes are perpendicular to latitudes. For spinning spheres, longitudes are along general direction of spherical axis, and latitudes are perpendicular to spherical axis.

Spherical coordinates can be vertical and horizontal latitude, so axes are perpendicular, but two latitudes define two different points, so points must have one more coordinate, such as north or south. Spherical coordinates can be vertical and horizontal longitudes, with axes not always perpendicular, but two longitudes can define the same great circle, so points must have one more coordinate. Therefore, only longitude and latitude define sphere points using two numbers.

MATH>Geometry>Solid>Sphere>Point

antipodes of sphere

Diameter intersects sphere at two points {antipodes, geometry}|.

pole

Spheres have points {pole, sphere} where meridians meet.

MATH>Geometry>Solid>Sphere>Spherical Angle

azimuth angle

Radii can make an angle {azimuth}| {polar angle} with polar axis.

spherical angle

Dihedral angles {spherical angle} are at diameter where great-circle planes intersect.

spherical degree

1/720 {spherical degree}| of sphere surface is solid-angle unit. One spherical degree is the birectangular spherical triangle whose third angle is one degree.

spherical excess

Difference {spherical excess} between spherical-polygon angle sum and plane angle sum is (n - 2) * (180 degrees), where n is number of angles.

steradian

Sphere has solid angle 4 * pi {steradian, solid angle}|.

MATH>Geometry>Solid>Sphere>Spherical Regions

cap of sphere

Spherical areas {cap, sphere} can go from pole down to circle where plane intersects sphere. Cap area is pi * d * h, where d is diameter, and h is cap height from center.

hemisphere

Sphere halves {hemisphere}| are solids.

lune

Two great circles not in perpendicular planes make two major and two minor spherical-surface regions {lune}.

segment of sphere

Sectors {segment, sphere} {major sector, sphere} {major segment, sphere} can be greater than hemisphere. Sectors {minor sector, sphere} {minor segment, sphere} can be less than hemisphere.

spherical wedge

Planes of two great circles intersect at diameter {spherical wedge} and divide sphere into four parts. Spherical-wedge volume is (A / (3 * pi / 2)) * pi * r^2, where A is angle between planes, and r is radius.

zone of sphere

Sphere and two parallel planes intersect to make a solid figure {zone, sphere region}. Sphere and plane intersection has area pi * d * h, where d is diameter, and h is height from center.

MATH>Geometry>Solid>Surface

development of solids

Solid surfaces can unfold or unroll so all faces or surfaces lie in one plane {development, solid}.

developable surface

Surfaces {developable surface} can flatten onto a plane without distortion, so surface line elements become plane line elements.

fixed point theorem

Continuous transformations of n-simplexes onto themselves have at least one fixed point {fixed point theorem}.

quadrature

Processes {quadrature} can try to find squares equal in area to surfaces. If plane figure has only straight lines, compass and straightedge can perform quadrature.

simplex

Spaces can have simplest geometric figure {simplex, space}| {space, cell}. Number of space dimensions defines simplex: 0 is point, 1 is line, 2 is triangle, 3 is tetrahedron, and n is n-simplex. Simplexes are manifolds. Simplexes have orientation. Even numbers of permutations make same orientation. Odd numbers of permutations make opposite orientation. Simplex boundaries are next-lower-dimension simplexes and have orientation.

square measure

Surface areas can use measures {square measure}.

trapezoidal rule

To find area under curve, replace curve with chords, to make equal-width orthogonal projections onto independent-variable axis, and add trapezoid areas {trapezoidal rule}|.

MATH>Geometry>Solid>Surface>Kinds

continuous surface

Surfaces {continuous surface} can have tangent plane and normal line at all points.

frustum

Truncated solids can have parallel plane sections {frustum}.

hyperboloid

Surfaces {hyperboloid} can have cross-sections that are hyperbolas.

isometric surface

If surfaces {isometric surface} bend without stretching and keep one-to-one correspondence, curvature and all other properties stay the same.

Kummer surface

Focal surfaces of systems of second-order rays are fourth-degree and class-four surfaces {Kummer surface}. Fresnel wave surfaces are special cases.

lamina surface

Solids can have plane parallel faces {lamina} that are small distance apart compared to face length.

paraboloid

Surfaces {paraboloid} can have sections that are parabolas. $x^2 / a^2 + y^2 / b^2 = 2*c*z$ {elliptic paraboloid}, where a b c are axes. Elliptic paraboloid with a = b is parabola rotated about its z-axis. $x^2 / a^2 - y^2 / b^2 = 2*c*z$ {hyperbolic paraboloid}, where a b c are axes.

ruled surface

Straight lines {rectilinear generator} can generate surfaces {ruled surface}| by translation in one direction, making lines at equal intervals. If there are two distinct generators, surfaces are doubly ruled. No generator can make skew surfaces.

smooth surface

Surfaces {smooth surface} can have no irregularities. Objects on smooth surfaces move only in direction tangent to surface.

unilateral surface

Surfaces {unilateral surface} can have one side.

MATH>Geometry>Space

space in geometry

Spaces {space, geometry} have elements. For example, space can be points. Elements combine to make space and figures in space. For example, points along one dimension form line, points along two dimensions form surface, and points along three dimensions form volume. Elements can be lines, surfaces, volumes, vectors, triangles, loops, colors, or objects. Points can represent vectors {vector space}. Phase-space elements can represent system states or dynamical equations.

dimension

Spaces have components {dimension}. Spaces can have zero-dimension points, one-dimension curves, two-dimension surfaces, and three-dimension solids. Spaces can have infinitely many dimensions and can have fractional dimensions.

component

Dimension can be infinite or finite. Finite dimensions can be circular.

continuous

Dimensions can be discrete or continuous.

elements

Space can have different elements, such as points or lines. Two-dimensional space can use three reference points. Three-dimensional space can use four reference points. Three-dimensional space can use three orthogonal lines or four non-orthogonal lines.

coordinates

In two-dimensional space, point position is (x,y) {ordered pair, space}, where x and y are distances to two reference lines. In three-dimensional space, point position is (x,y,z) {ordered triple}, where x, y, and z are distances to three reference lines.

direction

From reference points, lines can go in directions {direction} {orientation}.

elliptic space

Non-Euclidean space can have no parallel to given line through external point {elliptic space}. Elliptic space can have spherical geometry.

line segment

Line segments {line segment, magnitude} can represent magnitudes, ratios, or proportions.

MATH>Geometry>Transformation

transformation in space

Space coordinates or figures can move {transformation, space}.

translation

Motion can be along geodesic {translation, transformation}. Transformation shifts axes, keeping new axes parallel to old axes: x2 = x1 - h and y2 = y1 - k, where h and k are shift distances.

translation: shear

Translation can keep one coordinate axis or coordinate plane unchanged while others change {shear transformation} {shear translation}. Points move parallel to fixed axis or plane. Movement can be proportional to distance from fixed axis or plane.

dilation

Transformations {dilation transformation} can be through a fixed point, so distances from points to fixed point are a constant {constant of dilation} multiple of distances from fixed point to new points. Dilation is similar to similarity.

rotation

Rigid turning motion through angle can be around a common center or line {rotation, transformation}. Positive rotation is anti-clockwise. Negative rotation is clockwise. Transformation rotates both axes by angle A: x2 = x1*cos(A) + y1*sin(A) and y2 = - x1*sin(A) + y1*cos(A).

isometry

One-to-one transformation can leave distances, sizes, and shapes unchanged {isometry transformation}. Isometry involves translation and rotation. Rotate both axes by angle A and translate axes: x2 = x1*cos(A) + y1*sin(A) - h and y2 = - x1*sin(A) + y1*cos(A) - k, where h and k are shift distances.

reflection

Transformation can reflect both axes through origin: x2 = - x1 and y2 = - y2 {reflection, transformation}.

inversion

Transformation can involve both rotation and reflection {inversion, transformation}.

invariance

Transformations can result in same product as before {invariance, transformation}. Invariance example is geometric-figure rotation or reflection that transforms the points back into same figure {symmetry group, transformation}.

invariance: symmetry

For symmetric reflection through line or plane, if (x,y) is on figure, then (x,-y) or (-x,y) is on figure {axial symmetry transformation} {bilateral symmetry transformation}. For symmetric reflection through point, if (x,y) is on figure, then (-x,-y) is on figure {radial symmetry transformation} {point symmetry transformation}.

association

For three successive operations, find result of first and second and then do third, (a + b) + c, or do first then find result of second and third, a + (b + c) {association operation}. Results can be same product {associative}, (a + b) + c = a + (b + c), or different products {non-associative, transformation}, (a + b) + c != a + (b + c).

commutation

Two successive operations can happen in either order, a + b or b + a {commutation operation}. The result can be same product {commutative, transformation}, a + b = b + a, or different products {non-commutative, transformation}: a + b != b + a.

covariance

Transformations can result in same product as before but times constant.

contravariance

Transformations can result in same product as before, but using different coordinates.

group

Transformations form groups. Operations transform elements to other elements. In groups, transformations can reduce to other transformations or be irreducible. Metric determines possible coordinate transformations at manifold points. Transformations transform basis vectors into themselves, if transformation keeps same coordinate system. Transformations can transform basis vectors into their linear combinations.

affinities

Transformation groups can combine isometries, shear transformations, and similarities {affinities}.

birational transformation

Rational coordinate functions can have inverses {birational transformation}. Birational transforms can change irreducible algebraic plane curves to curves with no singular points, except for double points with distinct tangents.

continuity principle

If one figure derives from another by continuous changes, and derived figure has same generality as original figure, all first-figure properties are true of second figure {continuity principle} {correlativity principle} {contingent relations principle} {principle of continuity} {principle of correlativity} {principle of contingent relations}.

Cremma transformation

Derivatives of three-dimensional functions with spatial directions can be rational, single-valued, and solvable {Cremma transformation}. Cremma transforms can change irreducible algebraic plane curves to curves with no singular points, except for multiple points with distinct tangents.

duality in geometry

Lines and points are duals in plane projective geometry. For plane figures, true theorems {reciprocal theorem} about non-metric properties can interchange the words "line" and "point" {duality}| {principle of duality}. For example, "a unique line intersects two points" and "a unique point intersects two lines".

three dimensions

Planes and points are duals in three-dimensional projective geometry. For plane figures, true theorems about non-metric properties can interchange the words "plane" and "point", if the word "line" does not change.

metric

These duality principles are not true for metric properties.

homography transformation

Plane points and lines can transform {homography transformation} into points and lines in the plane or another plane, to make homologous or projectively related figures.

superposition axiom

Figures can move in space without shape or size distortion {superposition axiom}.

MATH>Geometry>Transformation>Symmetry

image of point

Point reflections make points {image, point}.

order of symmetry

Rotations around symmetry center can result in coincidence. Rotations can be fractions of 360 degrees: 180, 120, 90, 72, 60, 45, 30, and 15 degrees. 360 divided by rotation degrees makes whole number {order of symmetry}: 2, 3, 4, 5, 6, 8, 12, and 24.

MATH>Geometry>Transformation>Symmetry>Kinds

axial symmetry

For symmetry through line or axis {axial symmetry}|, (x,y) goes to (x,-y) or (-x,y).

radial symmetry

For symmetry through point or origin {radial symmetry}| {central symmetry} {point symmetry}, (x,y) goes to (x,-y), (-x,y), and (-x,-y).

MATH>Geometry>Kinds

differential geometry

Smooth manifolds allow any number of differentiations at all points {differential geometry}. Differential geometries use metrics and covariant derivatives. Riemannian geometry and pseudo-Riemannian geometry are differential geometries.

differential topology

Smooth manifolds have global differentiable functions {differential topology}. Differential topology uses no metric and no covariant derivatives.

ordered geometry

One point B can lie between two other points A and C {intermediacy}: ABC. All intermediate points B make a segment. All intermediate points B, plus points A and C, make an interval. All points past A in direction AC make a ray. An interval and its two rays make a line. Two rays with a common point make an angle.

Non-metric geometry {ordered geometry} can use intermediacy. Ordered geometry can be the basis of affine geometry and its sub-geometries: Euclidean geometry (parabolic geometry), absolute geometry, and hyperbolic geometry, because straight lines, parabolas, and hyperbolas are open figures and have between-ness. Ordered geometry cannot be the basis of projective geometry, because circles and ellipses are closed figures and so do not have between-ness.

MATH>Geometry>Kinds>Geometric Algebra

geometric algebra

Algebras {geometric algebra} can represent real and complex vector-space non-coordinate classical and relativistic geometries. Geometric-algebra elements (vectors) have dimension (grade), scalar amount (magnitude), space orientation/angle (direction), and relative direction {direction sense} (up or down, inside or outside, or positive or negative). Two vectors have a scalar dot product (inner product) {interior product}, bivector cross product (outer product), and inner product plus outer product (geometric product) (multivector). Grassmann and Clifford algebras generalize geometric algebra.

Clifford algebra

Hypernumber algebras {Clifford algebra} [1878] have 2^n dimensions for n number components. Dimensions represent reflections and rotations. Rotations are reflection combinations. Reflections convert right to left, or vice versa. Clifford algebras model spinors.

Grassmann algebra

In generalized geometric algebras {Grassmann algebra}, the basis elements are the unit-magnitude dimensions, which can be any number and can be non-orthogonal. Elements are dimension linear combinations and have grade, magnitude, direction, and direction sense.

Operations are reflections. Elements add to make a new element. Elements multiply to make an element of one higher dimension (wedge product) {Grassmann product, algebra}. Parallel vectors are commutative. Perpendicular vectors are anti-commutative. Elements are associative for addition and multiplication. Grassmann algebra [1844 and 1862] is Clifford algebra in which two successive reflections cancel, rather than making rotation, and so there are no rotations and no need for metric or perpendicularity.

MATH>Geometry>Kinds>Projective Geometry

projective geometry

Non-metric geometry {projective geometry}| can require that parallel lines meet at infinity {elliptic parallel axiom}: at a vanishing point on the horizon line. In the projective plane, two lines intersect at one point {elliptic incidence property}, so, for a line and a point not on the line, all lines through the point intersect the line once. In projective space, two planes intersect at one line.

Projective geometry involves constructions with straightedge only (no compass). Straight lines remain straight. Projective geometry is not about circles, angles, and parallels. All conic sections are equivalent.

Projective geometry has no circles, no angles, no measurements (no metric), no parallels, and no between-ness (no intermediacy).

Collinearity is invariant in projective geometry. Therefore, projective geometry has invariant point and line structures (incidence structure) (incidence relation) that make generalized planes. The Fano plane is the simplest 2-dimensional finite incidence structure.

Cross ratio is relation of projective harmonic conjugates. Cross ratios are invariant in projective geometry. Cross ratio is ratio of distance ratios. For four distinct collinear points A, B, C, and D in sequence, the cross ratio is (AC/BC)*(BD/AD) = (medium1/short)*(medium2/long) = (AC/BC) / (AD/BD) = (medium1/short)/(long/medium2) =

(AC*BD)/(BC*AD) = (13*24)/(23*14) = (medium1*medium2)/(short*long). For a line, if three points and the cross ratio are known, the fourth point is known.

Projective geometry has surface curve categories.

Projective geometry includes non-Riemannian geometry, single elliptic geometry, double elliptic geometry, and hyperbolic geometry.

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analytic surface

Spheres {analytic surface} are the only constant-positive-curvature closed surfaces with no singularities.

cross ratio

Ratios {cross ratio} can be distance from line-segment point to line-segment end divided by distance from line-segment point to other line-segment end. Cross ratio can apply in non-metric geometries. Cross ratio is relation of projective harmonic conjugates. Cross ratios are invariant in projective geometry. Cross ratio is ratio of distance ratios. For four distinct collinear points A, B, C, and D in sequence, the cross ratio is (AC/BC)*(BD/AD) = (medium1/short)*(medium2/long) = (AC/BC) / (AD/BD) = (medium1/short)/(long/medium2) = (AC*BD)/(BC*AD) = (13*24)/(23*14) = (medium1*medium2)/(short*long). For a line, if three points and the cross ratio are known, the fourth point is known.

Desargue theorem

If three line segments meet at point, they have one point each such that the three lines defined by line-segment endpoints intersect the three lines defined by the three pairs of points at three points on same line {Desargue's theorem} {Desargue theorem}.

harmonic section

For line segments, distance from a point to one end divided by distance from the point to other end {harmonic section} {harmonic ratio} equals negative of distance from one point {ideal point, line} on line-segment extension to one end divided by distance from ideal point to other end: PE1 / PE2 = - IE1 / IE2. For all other line-segment points, ratios {anharmonic ratio} are not harmonic. Point position determines ideal-point position {harmonic conjugate, endpoint}. If ideal point is at infinity, line-segment point is at midpoint. Projections keep harmonic ratio and relative positions the same.

incidence structure

Points lie on lines, lines include points, and points and lines have Cartesan point and line products {incidence structure} {incidence relation}. Projective geometry preserves incidence structure. Incidence structure is independent of intermediacy.

loxodrome

Mapping can project sphere onto plane and maintain constant bearings {rhumb line, loxodrome} {loxodrome}.

parallel axiom

A point is not on a line. In Euclidean geometry, parallel lines do not meet {parallel axiom}|, and only one line through the point is parallel to the line. In elliptic geometry, parallel great circles meet at two points, and only one line through the point is parallel to the line. Parallel great circles meet at poles of sphere. Parallel lines can make obtuse angles, as on imaginary-radius spheres, or acute angles, as on real-radius spheres. In hyperbolic geometry, parallel great circles do not meet, and many lines through the point are parallel to the line. In projective geometry, parallel lines meet at infinity.

Pascal theorem

Three sides of a conic-inscribed hexagon intersect at three points, which lie on same line {Pascal's theorem} {Pascal theorem}.

MATH>Geometry>Kinds>Projective Geometry>Kinds

algebra of throws

Projective geometry (Karl von Staudt) can use line-segment cross ratio and length analog {algebra of throws}. Lines (ray) can have cross-ratio coordinates 0 for one line-segment end, 1 for line-segment middle, and infinity for other line-segment end. Line segments can transform into arrays, and vice versa.

affine geometry

Non-metric geometry {affine geometry} can use parallel or cylindrical geometric projection, with projection center at infinity {ideal point at infinity}. Only one line goes through two points. Through a point not on a line, only one line is parallel to the line. Straight lines stay straight and parallel lines stay parallel, but lengths and angles can alter {non-Riemannian geometry}, so transformations and translations preserve parallel lines and distance ratios, but not necessarily congruence. Affine geometry is projective geometry with one line or plane as the points at infinity, which transformations leave invariant. (Projective geometry has only one projective plane.) Affine geometry is Euclidean geometry without congruence.

Ratios of separations are invariant in affine geometry. For three distinct collinear points A, B, and C in sequence, length ratio is AB/BC = medium1/medium2. Alternatively, AC/BC = long/medium2. Alternatively, AB/AC = medium2/long. For a line, if two points and the ratio of their separations are known, the third point is known.

Euclidean geometry

Space can have no curvature {Euclidean geometry}| {parabolic space} {parabolic metric geometry}. Geodesics are straight lines. Parabolic metric geometry preserves angle size and similar figures. Each line has one distinct real ideal point, and no line is perpendicular to itself. As conic sections, parabolas have no asymptotes and one distinct real ideal point.

postulates

Euclidean geometry has five assumptions {postulate}. Straight lines can go from any point to any other point. Finite straight lines can extend to any length and remain straight and in same direction. Circles have a center point and finite straight radius with one end in center. Two perpendicular straight lines intersect to make equal angles {right angle}. One and only one straight line through a point not on another straight line does not intersect second straight line {parallel postulate, Euclid}.

First two postulates do not state that lines are unique lines but use lines as if they are unique. Euclid's use of superposition without postulation is questionable.

axioms

Euclidean geometry has five facts that depend on reason or common sense {axioms, Euclid}. Things equal to same thing are equal to each other. If equal things add to equal things, sums are equal. If equal things subtract from equal things, differences are equal. Things that exactly overlap {coincide} are equal. Wholes are greater than or equal to any parts.

definitions

Euclidean geometry relies on definitions. Points are infinitely small space volumes. Two points define a line. Continuous infinite point sets can define lines. Lines have finite line segments. Lines have two infinite rays starting at a line point. Angles have two straight rays starting at vertex point.

Only Euclidean geometry, single elliptic geometry, double elliptic geometry, and hyperbolic geometry permit rigid figure motions.

hyperbolic geometry

In two dimensions, geometry can have saddle shape {hyperbolic geometry}| {Lobachevskian geometry, hyperbolic}. If space has hyperbolic geometry, infinitely many lines through a point not on a line are parallel to the line and do not intersect the line. If a shape becomes larger or smaller, shape changes, so figures cannot be similar.

Triangle angles add to less than pi. Pi minus angle sum varies directly with triangle area A: C * A = pi - (a + b + c), where C is Gaussian curvature. Gaussian curvature C = -1 / R^2, so for hyperbolic space, curvature radius is imaginary {pseudo-radius}.

As conic sections, hyperbolas have two asymptotes and two distinct real ideal points.

Hyperbolic surfaces can map conformally to discs in Euclidean planes {conformal model} {Poincaré model} {Poincaré disc}. Hyperbolic-surface straight lines are circle arcs that intersect disc edge at right angles. Hyperbolic-surface angles are the same as disc angles.

Hyperbolic surfaces can map non-conformally to discs in Euclidean planes {Klein representation}. Straight lines in hyperbolic surfaces are straight lines in discs. Angles in hyperbolic surfaces are not the same as angles in discs.

In discs, distances closer to edge contract more, and disc has bound. In hyperbolic surfaces, space has no boundary distances and does not contract. Distance between points A and B is 0.5 * ln((QA / QB) * (PB / PA)), where Q is closer to B and on disc and P is closer to A and on disc. Projective model is conformal model but with distances from center multiplied by (2 * R^2) / (R^2 + r^2), where R is disc radius, and r is distance of point from center. Conformal disc can project vertically onto hemisphere above disc {hemispheric representation}.

Lines can project from conformal circle {equatorial disc} to sphere south pole and to northern hemisphere {stereographic projection, geometry}.

Hyperbolic surface can project to Poincaré half-plane. Metric is (dx^2 + dy^2)/y^2.

Hyperbolic surface can project to surface with constant negative curvature {pseudo-sphere} {pseudosphere}. Distances in pseudo-sphere and Euclidean space are equal. To construct pseudo-sphere, rotate tractrix around its asymptote. To construct tractrix, use zero-rest-mass rod with large mass at one end resting on surface with friction and with other end on asymptote, then slide end along asymptote to trace curve with other end. Metrics 4 * (dx^2 + dy^2)/(1 - x^2 - y^2)^2 {Poincaré metric} can have constant negative curvature.

Special-relativity Minkowskian geometry is a hyperbolic space.

intrinsic geometry

Surface geometries {intrinsic geometry} need not use, or depend on, surrounding space or coordinates. Properties {intrinsic property} do not change if coordinates change. Equations {intrinsic equation} can use no coordinates. Intrinsic equations typically use only curvature radius and arc length.

metric geometry

Geometry {metric geometry}| can use magnitudes and measures. Euclidean geometry has a metric. Topology is non-metric geometry. In two dimensions, all metric geometries are projective geometry augmented by conic. In three dimensions, all metric geometries are projective geometry augmented by quadric {absolute curve}. Metric-geometry determinant must equal positive one or negative one.

Separation is invariant in Euclidean geometry. For two distinct collinear points A and B in sequence, separation is AB. For a line, if one point and the separation are known, the second point is known.

non-Euclidean geometry

Geometry {non-Euclidean geometry}| can be about curved spaces. Curved spaces can be hyperbolic spaces or spherical spaces.

single elliptic geometry

On spheres, great circles intersect at two points. So that two points determine a line, assume that opposite sphere points are identical {single elliptic geometry}, so two great circles intersect at only one point. For every great circle, through any point outside the great circle, no lines are parallel to the great circle. In triangles, angle sum is greater than 180 degrees. As conic sections, ellipses have no asymptotes and two imaginary ideal points.

spherical geometry

On spheres, great circles intersect at two points. Two points do not determine one line. For every great circle, through any point outside the great circle, no lines are parallel to the great circle {spherical geometry} {double elliptic geometry}. In triangles, angle sum is greater than 180 degrees. As conic sections, circles and ellipses have no asymptotes and two imaginary ideal points.