Outline of Game Theory
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MATH>Game Theory

game theory

Theories {decision theory, game} {operational research} {game theory}| can analyze competition and cooperation among players, who must make decisions that result in rewards, punishments, or neutral outcomes. Predictions require creating possible alternatives, stimulating system, extrapolating trends, and building relevance hierarchies to see how to reach goals.

decision

Decision requires thought, but choice does not. Decision selects option. Decision is not true or false. Decision can cause intention.

theory

People can minimize maximum risk {minimax, strategy} {maximin}, using decision values {statistical decision theory} {Wald theory}.

information in games

Games have possible player decisions and actions {information, games}, which have probabilities.

perfect information

Games can have no chance decisions {perfect information}. For perfect information, strategy determines outcome {normal form, games}. Strategies make decision-point action sequences {extensive form}.

Real games with perfect information often have too many decisions to record in normal form, so the game practically has imperfect information and has approximate strategy. Chess is two-person, zero-sum game with perfect information but very large decision space. 30,000 and 50,000 basic patterns exist. Mutual defense between pieces, cooperation in attacking common target, special pawn chains, and various castled-king patterns are basic patterns.

imperfect information

Most games involve chance {imperfect information}. Games can allow outside information. Life situations allow new action types.

player power

Players can control {power, player} {player power} game outcome or their gains to some extent.

strict determinacy

Games with perfect information {strict determinacy} have only required decisions and actions.

utility function

For all outcomes, sum of outcome value times outcome probability gives expected value or utility {utility function}. If games have known probabilities, as in gambling, and player outcomes have values, utility function is calculable.

MATH>Game Theory>Strategy

strategy

Principles or tables {strategy} determine decisions.

risk

Strategy {minimax theorem} can minimize risk. Minimax strategies are good, no matter what other player does. In finite, zero-sum, two-person game, one player gets average amount from the other if both use best strategy to minimize risk.

dominance

Strategy {dominant strategy} can always be better than or equal to all other strategies and is always better in at least one situation.

equilibrium

If one player maintains strategy, and the second changes strategy to best strategy, outcomes can stay constant {equilibrium point} {equilibrium strategy}. Games can have no equilibrium point. To try to maximize gain and minimize loss, use mixed strategies based on strategy-combination reward and punishment probabilities. Typical result is that no one gains or loses. If playing superior opponent, it is best to use mixed strategies.

Nash strategy

At least one strategy {Nash strategy} makes player better off if other players use best strategy. If all players use only Nash strategy, games play in set patterns in which all players are better off {Nash equilibrium}, but reward can be not optimum. In many-person games, many Nash equilibria exist, and no method can find best Nash strategy.

MATH>Game Theory>Coalition

coalition

In forming coalitions {coalition}, player power to affect outcome, including making coalitions, is the most-important factor. Player must gain equal or more value by being in coalition, or else player does not join group. Acceptable outcome requires Pareto optimum.

symmetry

All players can have same role {symmetric game}.

communication

Outcomes {discriminatory solution} can give less than fair share to outsider and keep rest for coalition, which typically happens when games allow communication.

set

Typically, each possible outcome dominates another outcome, and outcomes are intransitive, so no outcome dominates all others. Typically, many undominated outcomes result {imputation}.

An outcome {effective set} can dominate if players get more when in coalitions, and coalitions can form.

set: stability

Typically, at least one outcome in current set dominates all new outcomes, making pressure to stay in current set. If gains are large and number of people is large, stability is low.

Aumann-Masschler theory

Theories {Aumann-Masschler theory} can study game coalitions. Aumann-Masschler theory uses characteristic function form, determines outcomes for coalitions, does not try to predict coalitions or determine fairness or equity, assumes players are in only one coalition or act alone, and assumes coalitions have value equal to member-value sum. Aumann-Masschler theory uses no utility comparisons or Pareto optima.

objection

Games have bargaining sets. Players can want member to leave coalition, so remaining players can get more {objection}. Acceptable outcome has no objections or reciprocal objections.

characteristic function form

Games that reach situations with two coalitions {characteristic function form} are like two-person games.

self-policing set

Strong imputations can cause dominating outcomes {self-policing set} outside current set to cause coalition members to take losses.

Shapley value

For all players, values {Shapley value} can measure difference between coalition utilities if player is in or out of coalition. All coalitions have Shapley values. Coalition closest to having half total power is most likely. Coalition must have at least half total power to determine outcome. In coalitions, strong players gain more than weak players, but strong-weak coalitions do not form often. Players do best in only one coalition.

superadditivity

Coalition value can equal or exceed {superadditivity} coalition-player-value sum, because utility can transfer.

MATH>Game Theory>Communication

communication in games

Games can allow threats, binding or non-binding agreements, side payments, real and pretended restrictions, and real or false information {communication, games}. Communication can be free or restricted by rules. Rules and normal form have equal importance, depending on ability to communicate and cooperate. Games can allow shared rewards. Games can allow binding or non-binding agreements among communicating players. Agreements can be free or restricted by rules.

Nash arbitration scheme

Arbitration {Nash arbitration scheme} can maximize player-utility product.

Pareto optimum in games

Arbitrators and bargainers try to find point at which both players are best off {Pareto optimum, games}. Typically, more than Pareto optimum exists.

MATH>Game Theory>Communication>Bargaining

bargaining

Bargaining {bargaining, games} {negotiation, arbitration}, or using third party to determine outcome fairly {arbitration, negotiation}, makes outcome independent of utility function, as well as independent of allowing other outcomes. In cooperative games and bargaining, one player often dominates.

bargaining set

Games have acceptable-outcome sets {bargaining set}.

Bowley point

In bargaining, player can assume other player will maximize outcome {Bowley point}. Games with no repeated play and incomplete information often reach the Bowley point, instead of Pareto optimum.

equal profits point

Bargaining can find point where outcomes are equal for both players {equal profits point}.

MATH>Game Theory>Game Types

game types

Games have kinds {game types}. In real games, players often value rewards differently than in simulated games. For example, rather than using strategy that tries to minimize losses, players often use strategy that tries to get greatest average return. Players tend to cooperate more. Personal or emotional factors, such as ideology, can affect decisions. Player goals determine decisions.

In real n-person games, communication difficulties are important, player physical arrangement is important, and aggressive and fast-acting players typically do better. People typically do not maintain utility transitivity, so value assignments are inconsistent. Environment can change and affect values and probabilities used in utility function. Players typically do not gamble large amounts, and they gamble based on emotion, not calculated utility.

Colonel Blotto game

Two-person, finite, zero-sum games {Colonel Blotto game} can involve one decision from each player.

compound game

Games {compound game} can have several independent games. Compound-game expected value is sum of component-game expected values.

finite game

Most games {finite game} have limited move numbers. Few games allow infinite numbers of moves.

non-zero-sum game

Different outcomes can have different total rewards and punishments {non-zero-sum game}|, so both players can gain or lose.

one-person game

Games {one-person game} can have one player.

types

In one-person game, player decisions alone can determine outcome, with no decisions determined by chance, as with Tower of Hanoi. Alternatively, player decisions and decisions with known probabilities, as with dice games, determine outcome. Alternatively, player decisions and unknown-probability decisions, such as acts of god, determine outcome.

strategy

In one-person games, player typically assumes worst outcome will happen and uses minimax strategy.

prisoner's dilemma

In two-person, non-zero-sum games {prisoner's dilemma}|, players can choose from two alternatives. If both players choose number one, both players get reward. If both players choose number two, both players get punishment. If one player chooses number one and the other chooses number two, first player has more punishment, but second player has more reward.

strategy

Both players do best if they cooperate. However, if one player uses only one strategy, other player can change strategy and get big reward.

experiments

In real experiments, finite games have no cooperation, but infinite games have cooperation.

factors

Reward and punishment sizes, previous play, communication ability, and personality affect the game, so it is a competitive game.

traveler's dilemma

In a two-person, non-zero-sum game {traveler's dilemma}|, players can choose number from 2 to 100. Third party asks each to state number. If both write same number, they both receive that amount. Otherwise, lower number receives that amount plus bonus, and higher number receives that amount minus penalty.

two-person game

Games {two-person game} can have two players.

zero-sum game

Games {zero-sum game}| can have total rewards and punishments that are the same for all outcomes, so players can gain only at expense of other players.

examples

Chess, checkers, go, and tic-tac-toe are zero-sum games, because they are win, lose, or draw.

strategy

Two-person, finite, zero-sum games have strategy that wins for first player, makes first player lose, makes it so first player cannot lose, or makes first player lose or draw.

types

Typical two-person, finite, zero-sum game, such as rock-scissors-hand, has no perfect information, because players choose simultaneously.

non-zero-sum

Different outcomes can have different total rewards and punishments, so players can gain or lose, in a non-zero-sum game, as in prisoner's dilemma.

MATH>Game Theory>Voting

voting

People can cast ballots {voting}| to determine election or referendum. Voting chooses among alternatives, perhaps allowing write-in candidates. Winner is choice with the most votes. Jurisdictions can require people to vote or restrict voting by literacy.

types

Voting can use different systems with different outcomes. From list, people can choose one alternative. If more than half the voters select one choice, it wins. If no choice has more than half the vote, choice with most votes can win, or runoff can select between the two choices with most votes. People can indicate preference order {proportional voting}. Vote counting weights preference order and selects most popular overall.

Condorcet paradox

Simple majority rule can violate transitivity {Condorcet paradox, voting}, because rankings can permute in all ways {Condorcet cycle, voting}.

democratic voting

Democratic decisions can result if five assumptions hold {democratic voting}. With three or more candidates or proposed laws, the five assumptions are inconsistent.

single-peaked preference

Voters can prefer only one outcome {single-peaked preference}. With single-peaked preferences, voters choose outcome closest to preference, group preferences are transitive, and outcome is acceptable to all voters, because it is near middle.

social welfare function

If ballot has several alternatives, several voters will vote, voters have different preferences between any two alternatives, and outcome depends on more than one player, no method can find true group preference {social welfare function}. Preferences among some alternatives are independent of preferences for others, and if more people favor alternative, it stays favored {Arrow social welfare theorem}.

MATH>Game Theory>Voting>Attitude

sincere voting

Voting rounds {sincere voting} can indicate people's true relative preferences.

strategic voting

Voting rounds {strategic voting} can invoke strategy to help people's true preference win.

MATH>Game Theory>Voting>Methods

instant-runoff voting

Voters can indicate preference order. Count first eliminates lowest-ranked candidate and apportions that candidate's votes to remaining candidates. Count then eliminates lowest-ranked candidate and apportions that candidate's votes to remaining candidates, and so on, until only one candidate remains {instant-runoff voting} (IRV).

plurality voting

Candidate or proposition can win with the most votes {plurality voting}, even with no majority. Allowing plurality voting tends to merge parties, and voters tend to avoid extremes, so outcomes tend toward middle.

proportional representation

Number of representatives for a group can depend on group vote percentage {proportional representation}. This method allows more extremes, such as fringe parties. Under proportional representation, small groups that vote together usually have more power than large disorganized groups.

rank-order voting

In one voting round, people assign rank, from 1 to candidate number, to all candidates {rank-order voting} {Borda count}. Winner has the most points. Because it is numerical, it can violate the neutrality principle.

successive procedure

Voting methods can first decide between two choices {successive procedure}, then decide between winner and third choice, then decide between winner and fourth choice, and so on. Final winner wins.

true majority rule

In one voting round, people list preference order among candidates {true majority rule} {simple majority rule}. Winner has won all one-on-one contests.

MATH>Game Theory>Voting>Principles

equal treatment principle

Voters have equal weight {equal treatment principle} {one-person one-vote principle} {anonymity principle}.

neutrality principle

Election system gives no advantage to any candidate {symmetry principle} {neutrality principle}. Third-candidate presence or absence can have no affect on choice between two candidates. Third-candidate preference change can have no affect on outcome.

Pareto principle

With several candidates, someone who always ranks below another person must not defeat that person {Pareto principle}.

transitivity principle

If voter prefers candidate over second one, whom he or she prefers over third one, voter prefers first one over third one {transitivity principle}. In simple majority rule, if there are three candidates, and one is never second {value restriction}, transitivity holds. In simple majority rule, transitivity holds if all voters base decision on one parameter. Simple majority rule can violate transitivity {Condorcet paradox, transitivity} because rankings can permute in all ways {Condorcet cycle, transitivity}. In case of no simple majority, using rank-order to supplement vote maintains transitivity.