Outline of Calculus
May 11, 2013

Contents

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MATH>Calculus

calculus math

Calculus can calculate quantities when other quantities are changing {calculus, mathematics}. For example, distance equals velocity times time, but velocity can change as time changes. Circle, ellipse, parabola, and hyperbola slopes change as position changes.

exterior calculus

P-forms have derivatives {exterior derivative}. Integral of p-form exterior derivative over region equals integral of p-form over region boundary {exterior calculus fundamental theorem} {fundamental theorem of exterior calculus} {exterior calculus}. Generally, integral of p+1-form over p dimension compact oriented region equals integral of p-form over p+1 dimension compact oriented region. Boundary of boundary is 0.

fundamental theorem of calculus

Integration inverts differentiation, and differentiation inverts integration {fundamental theorem of calculus}. Integral of f'(x) * dx = f(x). d(integral of f(x) * dx) / dx = f(x). If f(x) = g'(x), integral from a to b of f(x) * dx equals g(b) - g(a).

MATH>Calculus>Differentiation

differentiation in calculus

For continuous functions, range has change rate {derivative, function} {first derivative} with domain {differentiation, mathematics}. Functions f have independent-variable domain, such as time t, and dependent-variable range, such as distance x: f(t) = x. You can differentiate to find how distance x varies with time t: velocity v = df = dx / dt. For functions whose domain is time and whose range is velocity, you can differentiate to find how velocity v varies with time t: acceleration a = dv / dt.

Curve or surface functions have y-axis range and x-axis domain. At domain and range points (x,y), you can differentiate to find angle A of tangent to curve: tan(A) = dy / dx. You can also calculate slope of line normal to curve or surface.

See Figure 1. For function y = f(x) and two function points (x1, y1) and (x2, y2), change rate {slope, function} between two points is (y2 - y1) / (x2 - x1). Slope converges to value {limiting value} {limit} as x2 - x1 approaches zero at point (x1, y1). Limit is change rate at point (x1, y1).

See Figure 2. Functions have maximum or minimum where tangent slope is zero, because function has reached top or bottom.

Figure 1

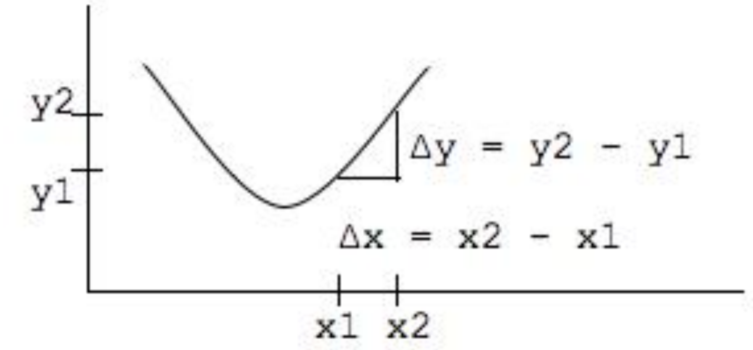


Figure 2

limit theorem

At a point, if two functions have limits, function-sum limit is sum of function limits {limit theorem}. Function-product limit is product of function limits. Function-quotient limit is quotient of function limits. For functions, nth-root limit is nth root of limit.

non-decreasing function

Functions {non-decreasing function} can have derivatives >= 0 over intervals.

non-increasing function

Functions {non-increasing function} can have derivatives <= 0 over intervals.

second derivative

Functions can have derivatives {second derivative} of first derivatives. Second differential is d2x [2 is superscript] or d2f(x): (d^2)x or (d^2)f(x).

MATH>Calculus>Differentiation>Difference

difference operator

For discontinuous functions, operator {difference operator} can find finite difference between (n+1)th term and nth term: y(n + 1) - y(n). First-order difference operator symbol is uppercase Greek letter delta. Second-order difference operator symbol is uppercase Greek letter delta squared.

differential in mathematics

Vanishingly small increment or infinitesimally small interval can have symbol dx {differential}. dx = (x + delta_x) - x, as delta_x approaches zero. For function, df(x) = f(x + delta_x) - f(x), as delta_x approaches zero. Therefore, (f(x + dx) - f(x)) / ((x + dx) - x) ~ df(x) / dx or f'(x) and f(x + dx) ~ f(x) + f'(x) * dx ~ f(x) + df(x).

exact differential

Variable partial derivative times variable differential is variable change {exact differential}: (Df(x,y) / Dx) * dx = (x change), where f(x,y) is a two-variable function, D is partial derivative, and dx is differential. For all variables, sum exact differentials. For two variables, (Df(x,y) / Dx) * dx + (Df(x,y) / Dy) * dy = (x change) + (y change). First-order differential equation can use differentials.

MATH>Calculus>Differentiation>Slope

directional derivative

At point, in direction, surface has slope {directional derivative}|. Directional-derivative vector is tangent to manifold in direction.

rate of change

Function derivatives {rate, differentiation} can be with respect to time.

singular point

Curve gradient can be indeterminate at point {singular point, curve}.

MATH>Calculus>Differentiation>Optima

minimum of curve

Relative minima {minimum, curve} have first derivative zero and second derivative greater than zero.

maximum of curve

Relative maxima {maximum, curve} have first derivative zero and second derivative less than zero.

concave curve

Functions {concave curve} can have points where second derivative is greater than zero {concave upward, concave curve} {convex downward, concave curve} and tangent is below curve. Functions can have points where second derivative is less than zero {concave downward, concave curve} {convex upward, concave curve} and tangent is above curve.

convex curve

Functions {convex curve} can have points where second derivative is greater than zero {concave upward, convex curve} {convex downward, convex curve} and tangent is below curve. Functions can have points where second derivative is less than zero {concave downward, convex curve} {convex upward, convex curve} and tangent is above curve.

optima

Functions can have points {optimum, calculus} {optima} where they are greatest or least. Functions can have highest points {relative maximum} in intervals. Functions can have lowest points {relative minimum} in intervals.

derivative

At maximum or minimum, derivative equals zero or has no definition. If first derivative changes sign, point is relative maximum or minimum.

Two-variable function maximum and minima are at points where both partial derivatives are zero. Maximum is if second partial derivative with respect to variable is less than zero: D^2f(x,y) / Dx < 0, where D^2 is second partial derivative, D is partial derivative, x and y are variables, and f is function. Minimum is if second partial derivative with respect to variable is greater than zero: D^2f(x,y) / Dx > 0.

Product of second partial derivatives with respect to each variable minus product of second derivatives with respect to each variable must be greater than zero: (D^2f(x,y) / Dx) * (D^2f(x,y) / Dy) - (d^2f(x,y) / dx^2) * (d^2f(x,y) / dy^2) > 0, where D^2 is second partial derivative, D is partial derivative, d^2 is second derivative, d is derivative, x and y are variables, and f is function.

inflection point

Function can have points {inflection point}| where second derivative equals zero, tangent intersects curve, and curve has zero curvature. At inflection point, curve changes from concave to convex, or vice versa.

MATH>Calculus>Differentiation>Mean Value

mean value theorem

If a continuous function has derivatives at all interval points, at one or more points derivative has slope equal to slope of straight line passing through interval endpoints {mean value theorem}. For interval (a,b), line slope is (f(b) - f(a)) / (b - a). Point (x,y) has a <= x <= b and dy/dx = (f(b) - f(a)) / (b - a).

extended law of the mean

For two functions over interval, at one or more points derivative ratio equals difference ratio {extended law of the mean} {law of the mean extended}. For interval (a,b), and functions f(x) and g(x), at (x,y), (f(b) - f(a)) / (g(b) - g(a)) = f'(x) / g'(x).

Rolle theorem

If continuous function has two roots over interval and has derivatives at all interval points, first derivative equals zero at one or more points {Rolle's theorem} {Rolle theorem}.

MATH>Calculus>Differentiation>Partial

partial derivative

Function can have two or more independent variables. To find derivative with respect to variable {partial derivative}|, hold other variables constant: df(x, y, z, ...) / dx = df(x, a, b, c, ...) / dx, where d is derivative, f is function, xyz are variables, and abc are constants.

change

For two variables, function-change slope or gradient df equals function value at x + dx and y + dx minus function value at x and y: df = f(x + dx, y + dy) - f(x,y) = (Df(x,y) / Dy) * dx + (Df(x,y) / Dy) * dy = D^2f(x,y) / DxDy, where d is differential, D is partial derivative, and D^2 is second partial derivative. Total change is sum of x and y changes. Direction change is partial derivative.

order

Order of taking partial derivatives does not matter, because variables are independent.

complex numbers

Complex-number differential depends on Cartesian differential. M is complex-number real part and N is imaginary part. dx and dy are infinitesimals on x-axis and y-axis. dq and dp are infinitesimals on real and imaginary axes. dq = M * dx + N * dy and dp = N * dx - M * dy. Therefore, Dp / Dx = Dq / Dy and Dp / Dy = - Dq / Dx {Cauchy-Riemann equations, partial derivative}, where D denotes partial differentials.

implicit function

At point, two-variable function {implicit function, differentiation} can equal zero: f(x,y) = 0. Then, derivative of one variable by other variable, dy/dx, equals negative of function partial derivative with respect to x divided by function partial derivative with respect to y, because differentiation makes constant zero: dy/dx = -(Df(x,y) / Dx) / (Df(x,y) / Dy), where D is partial derivative.

Because differentiation makes constant zero, constant can be any value. Constant can be implicit-function-family {primitive function} parameter. Differential equation is sum of parameter equation and primitive function.

MATH>Calculus>Differentiation>Function

constant differentiation

For constants {constant, differentiation}, function does not change, so derivative is zero. f(x) = c, and df(x) / dx = 0.

constant times function

For constant times function {constant times function differentiation}, derivative is constant times function derivative. y = c * f(x), so dy / dx = c * df(x) / dx.

power function differentiation

For power functions {power function differentiation}, reduce exponent by one and multiply by original exponent: dx^n = n * x^(n - 1) * dx. For example y = x^3, dy / dx = 3 * x^2.

MATH>Calculus>Differentiation>Function>Exponential

exponential function differentiation

d(e^x) / dx = e^x {exponential function, differentiation}. d(e^u(x)) = e^u(x) * du(x). Limit of (1 + 1/n)^n is e. Limit of (1 + h)^(1/h) is e. Limit of (1 + dx)^(1/dx) is e. Limit of (1 + (x / dx))^(x / dx) is e. On semi-log graph paper, y = b * a^x makes straight lines. On log-log graph paper, y = b * x^a makes straight lines.

logarithmic function differentiation

If x > 0, dln(x) = 1/x {logarithmic function, differentiation}. If u(x) > 0, dln(u(x)) = (1 / u(x)) * du(x). (v(x))^a = e^(a * ln(v(x))). (v(x))^z(x) = e^(z(x) * ln(v(x))).

MATH>Calculus>Differentiation>Function>Trigonometric

trigonometric function differentiation

dsin(x) = cos(x) {trigonometric function, differentiation}. dcos(x) = -sin(x). dtan(x) = (sec(x))^2. dcot(x) = -(csc(x))^2. dsec(x) = sec(x) * tan(x). dcsc(x) = -csc(x) * cot(x).

trigonometric function inverse differentiation

darcsin(x) = 1 / (1 - x^2)^0.5 {trigonometric function, inverse differentiation}. darccos(x) = -1 / (1 - x^2)^0.5. darctan(x) = 1 / (1 + x^2). darccot(x) = -1 / (1 + x^2). darcsec(x) = 1 / (x * (x^2 - 1)^0.5). darccsc(x) = -1 / (x * (x^2 - 1)^0.5).

hyperbolic function differentiation

sinh(x) = (e^x - e^-x) / 2 and cosh(x) = (e^x + e^-x) / 2 {hyperbolic function, differentiation}. dsinh(x) = cosh(x). dcosh(x) = sinh(x).

MATH>Calculus>Differentiation>Function>Multiple

sum of terms differentiation

For sum of terms {sum of terms differentiation}, find sum of differentials. For h(x) = f(x) + g(x), dh(x) = df(x) + dg(x).

product of functions differentiation

For product of functions {product of functions differentiation}, add second function times first-function differential and first function times second-function differential: g(x) * df(x) + f(x) * dg(x).

quotient of functions differentiation

For quotient of functions {quotient of functions differentiation}, multiply second function by first-function differential: g(x) * df(x). Then subtract first function times second-function differential: g(x) * df(x) - f(x) * dg(x). Then divide by second function squared: (g(x) * df(x) - f(x) * dg(x)) / (g(x))^2.

MATH>Calculus>Differentiation>Function>Vector

gradient of vector

Vector functions {gradient, vector}| can be sum of each partial derivative times its unit vector i: (Df(x,y) / Dx) * i + (Df(x,y) / Dy) * j, where D is partial derivative. Gradient is in respect to direction. Gradient uses an operator {del operator}, which is upside-down uppercase delta: del = ((D / Dx) * i + (D / Dy) * j). For two dimensions, gradient is normal vector to vector-function curve. For three dimensions, gradient is normal vector to vector-function surface.

curl of vector

Vector functions {curl, vector}| can be vector products of del operator and vector function: del x f. Curl of gradient of scalar function equals zero: del x (del f) = 0.

divergence of vector

Scalar functions {divergence, vector}| can be scalar products of del operator and vector function: del . f. Divergence of curl equals zero: del . (del x f) = 0.

MATH>Calculus>Differentiation>Methods

finite differences method

For f(x) near x = a, f(a + h) - f(a) = (h/c) * C + (h / (2*c)) * (h/c - 1) * C^2 + ..., where C = f(a + c) - f(a), c = x change, and h = x - a {method of finite differences} {finite differences method}.

Leibniz theorem

Rules can find nth derivative of function product {Leibniz's theorem} {Leibniz theorem}. If w = u*v, (D^n)w = (D^n)u * v + ... + (D^(n/2))u * (D^(n/2))v + ... + u * (D^n)v, where D^n is nth partial derivative and D^(n/2) is (n/2)th partial derivative. The rule makes binomial expansion series with n + 1 terms.

L'Hospital rules

Limit of function ratio is limit of function first-derivative ratio, if function limit equals zero or if denominator-function limit is positive infinity or negative infinity {L'Hôspital's rules} {L'Hôspital rules}.

MATH>Calculus>Differentiation>Methods>Chain Rule

chain rule

For functions of functions {function of function differentiation}, multiply differential of main function and differential of other function: dg(f) = (dg / df) * df {chain rule, differentiation}. Other function can be independent variable: df(x) = (df(x) / dx) * dx.

chain rule for partial derivatives

Derivative of two-variable function f(x,y) with respect to variable t is (Df(x,y) / Dx) * (dx/dt) + (Df(x,y) / Dy) * (dy/dt), where D is partial derivative {chain rule, partial derivatives}.

MATH>Calculus>Integration

integration in calculus

For smoothly changing continuous functions {integrand}, with y-axis range and x-axis domain, you can calculate area between curve and x-axis {integral} {first integral} {integration, calculus}, from lower domain value {lower limit} to higher domain value {upper limit}. For example, you can calculate enclosed-surface area and volume.

summation

See Figure 1. Domain goes continuously from lower value x1 to higher value x2, while range goes continuously from lower value y1 to higher value y2. Length x2 - x1 can divide into number N of intervals with equal widths (x2 - x1)/N.

In Figure 1, dashed line divides x2 - x1 interval into two intervals, each with width (x2 - x1)/2, because N = 2.

Range at left of each small interval is f(x1 + (ni - 1) * (x2 - x1)/N), for ith interval. For i = 1, it is f(x1). For i = 2, it is f(x1 + (x2 - x1)/2). Range halfway between left and right of each small interval is f(x1 + (ni - 1/2) * (x2 - x1)/N). For i = 1, it is f(x1 + -(1/2) * (x2 - x1)/2). For i = 2, it is f(x1 + (3/2) * (x2 - x1)/2). Range at right of each small interval is f(x1 + ni * (x2 - x1)/N). For i = 1, it is f(x1 + (x2 - x1)/2). For i = 2, it is f(x1 + 2 * (x2 - x1)/2). Product of range and interval width is rectangular area. For example, using left of each small interval, (f(x1 + (ni - 1) * (x2 - x1)/N)) * (x2 - x1)/N, for ith interval. For i = 1, it is f(x1) * (x2 - x1)/2. For i = 2, it is (f(x1 + (x2 - x1)/2)) * (x2 - x1)/2. Sum of all interval areas approximates total area between curve and x-axis, between domain values. As interval number increases, widths decrease, and area sum approaches true area.

interval position

The three different ways of taking interval range do not matter, because total area is same. For example, using right of each small interval, (f(x1 + ni * (x2 - x1)/N)) * (x2 - x1)/N, for ith interval. For i = 1, it is (f(x1 + (x2 - x1))/2) * (x2 - x1)/2. For i = 2, it is f(x1) * (x2 - x1)/2. Total area is the same.

number of intervals

Number of intervals does not matter. Use function f(x) = x^2, as in parabola. Interval is x1 = 0 to x2 = b. Number of subintervals is N = 3. (x2 - x1)/N = b/3. If f(x) is at midpoint of each interval, sum from x1 = 0 to x2 = b of f(x1 + (ni - 1/2) * (x2 - x1)/N) * (x2 - x1)/N is ((b/6)^2 + (b/2)^2 + ((5 * b)/6)^2) * (b/3), which is (b^3)/3. If function f(x) = x^2, x1 = 0, x2 = b, and N = 6, sum from x1 = 0 to x2 = b is ((b/12)^2 + ((3 * b)/12)^2 + ((5 * b)/12)^2 + ((7 * b)/12)^2 + ((9 * b)/12)^2 + ((11 * b)/12)^2) * (b/6), which is (b^3)/3. Therefore, results for different numbers of intervals are the same.

line

If f(x) = x, function is line. See Figure 2. Sum from x1 = a to x2 = b with N = 1 of f(x1 + (ni - 1/2) * (x2 - x1)/N) * (x2 - x1)/N is ((b + a)/2) * (b - a), which is area of trapezoid of base b - a and heights a and b.

constant

If f(x) = C, function is constant. See Figure 3. Sum from x = b to x = a with N = 1 of f(x1 + (ni - 1/2) * (x2 - x1)/N) * (x2 - x1)/N is C * (b - a), which is area of rectangle with height C and length b - a.

definite integral

Given function f(x) and interval (b - a), you can calculate integral from x = a to x = b of f(x) * dx {definite integral}. Domain-value variable dx {dummy variable} {variable of integration} does not appear in definite-integral result, because domain values over interval replace it.

indefinite integral

Without using interval, formula or other method can calculate integral {indefinite integral} {anti-differential} {antiderivative} {antiderived function}. Antiderivatives are functions from which original function can derive by differentiation.

Because derivatives of constants equal zero, function antiderivatives differ by a constant {constant of integration}. Knowing original-function domain and range allows calculating constant.

Figure 1

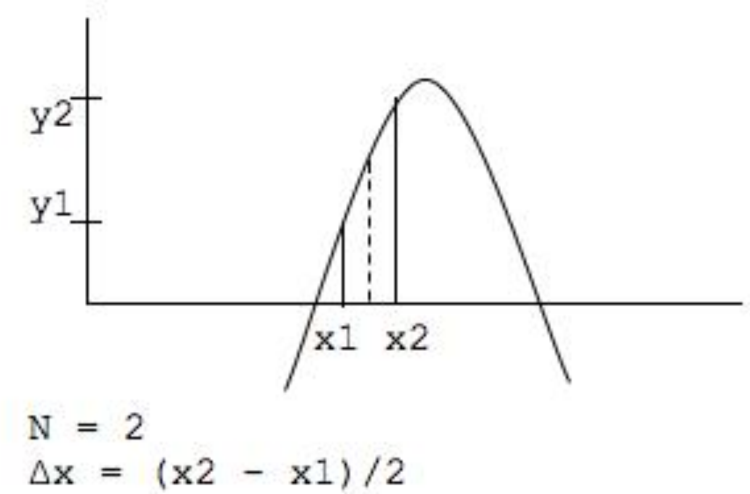


Figure 2

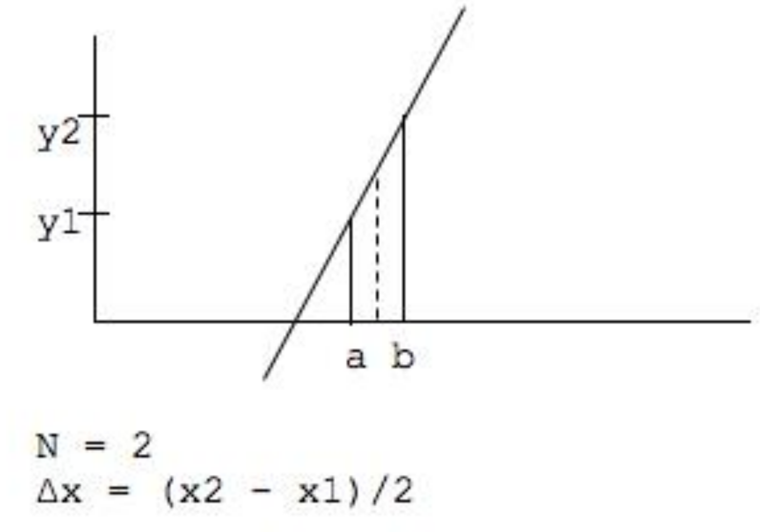
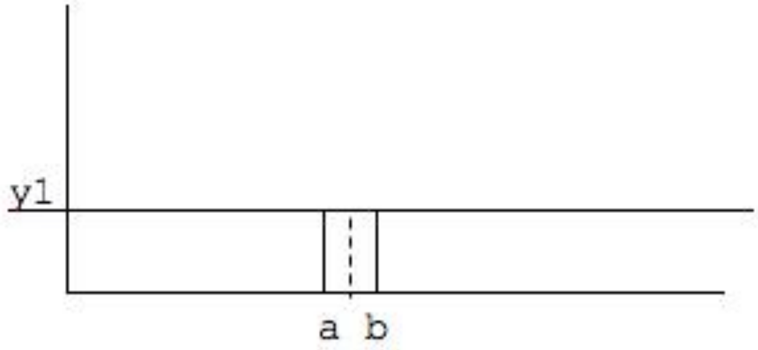


Figure 3



$N = 2$

$\Delta x = (x2 - x1)/2$

$y = y1$

hyperelliptic integral

Degree-greater-than-four integrals {hyperelliptic integral} can be rational elliptic-integral products.

integral equation

Equations {integral equation} can represent an infinite number of ordinary differential equations.

MATH>Calculus>Integration>Double

double integral

For two-independent-variable functions, calculate integral {double integral} by holding first variable constant and integrating over second variable, then holding second variable constant and integrating over first variable, and then adding results. It does not matter which variable is first. Double-integral domain can be surface {closed region} inside closed curve.

area by integration

To find surface area {area, integration}, take double integral over differential area (1 + (df(x,y) / dx)^2 + (df(x,y) / dy)^2)^0.5 * dx * dy.

divergence theorem

Triple integral, over volume, of scalar product of del operator and vector function equals double integral, over surface, of scalar product of function and normal vector to surface {divergence theorem}.

Cavalieri theorem

If two solids have equal altitudes and all sections parallel to base have same ratio, volumes have same ratio {Cavalieri's theorem} {Cavalieri theorem}.

Laplacian

The gradient of scalar electric potential is vector electric field. A scalar function has a divergence of the gradient {Laplacian}: D^2f/Dx^2 + D^2f/Dy^2 + D^2f/Dz^2, where D is partial derivative. Potential relates to charge density as Laplacian of potential equals negative of charge density divided by electrostatic constant (Poisson's equation). If charge density is zero, Laplacian of potential equals zero (Laplace's equation).

Schwarz paradox

Curved-surface area is not the limit of surface's plane-triangle areas {Schwarz's paradox} {Schwarz paradox}.

Dirichlet principle

In a plane region, if function has no singularities, is single-valued, and solves the potential equation, double integral of ((du/dx)^2 + (du/dy)^2) * dx * dy over x and y has a minimum {Dirichlet principle} {Thomson principle}.

MATH>Calculus>Integration>Line

line integral

Integral between two curve points is integral {line integral} {curvilinear integral} from a to b of p(x, f(x)) * dx, where f(x) is curve function, and p is surface function.

Green theorem

For closed curve, line integral over line equals double integral over closed region {Green's theorem} {Green theorem}. Line integral over regular simply connected closed curve equals zero. Line integrals over any two regular curves between two region points are equal. For closed surface, double integral over surface equals triple integral over closed volume.

Stokes theorem

Double integral, over surface, of cross product of del operator and vector function, equals line integral, over boundary curve, of vector function {Stokes theorem}.

MATH>Calculus>Integration>Line>Complex

contour integration

In complex plane, paths {closed contour} can loop around origin. There is line integral {contour integration} around the path. Looping once increases integral by 2 * pi * i. Looping counterclockwise once increases integral by -2 * pi * i.

Cauchy integral theorem

Integration paths over complex functions do not matter {Cauchy integral theorem}. For complex function f(z), integral of f(z) = F(z) = (1 / (2 * pi)) * (integral from p = -pi to p = +pi of (x * f(x) / (x - z)) * dp), where z is complex number, and x = |z| * e^(i*p) {Cauchy integral formula}. Cauchy integral formula makes series {majorant series} and has residue {integral residue}.

MATH>Calculus>Integration>Function

constant times function integration

To integrate constant times function {constant times function integration}, take integral of k * u(x) * dx = k * (integral of u(x) * dx), where k is constant, and u(x) is function.

linear function integration

Integral from a to b of f(x) * dx equals f(mean) * (b - a) {linear function, integration}. Integral of sum equals sum of integrals.

power function integration

To integrate power function {power function, integration}: Increase exponent by one and divide by new exponent. Integral of x^p * dx = x^(p + 1) / (p + 1), if p != -1 (p <> -1), so integral of x^3 = x^4 / 4. Integral from x = 1 to x = b of (1/x) * dx equals ln(b).

exponential function integration

Integral of e^x = e^x {exponential function, integration}. Integral of b^x = (1 / ln(b)) * b^x.

logarithmic function integration

Integral of ln(e^x) = integral of x {logarithmic function, integral}.

midpoint rule

For degree 1, 2, or 3 polynomials P(x), definite integral over interval (a,b) is ((b - a) / 6) * (P(a) + P(a + b) / 2 + P(b)) {midpoint rule}.

trigonometric function integration

Integral of sin(x) = -cos(x) {trigonometric function, integration}. Integral of cos(x) = sin(x). Integral of tan(x) = -ln(|cos(x)|). Integral of cot(x) = ln(|sin(x)|). Integral of sec(x) = ln(|sec(x) + tan(x)|). Integral of csc(x) = ln(|csc(x) - cot(x)|). Integral of (sin(x))^2 = (x - sin(x) * cos(x)) / 2. Integral of (cos(x))^2 = (x + sin(x) * cos(x)) / 2. Integral of (tan(x))^2 = tan(x) - x. Integral of (cot(x))^2 = -cot(x) - x. Integral of (sec(x))^2 = tan(x). Integral of (csc(x))^2 = -cot(x).

MATH>Calculus>Integration>Function>Multiple

ratios of polynomials integration

If numerator polynomial has higher degree, divide polynomials to get quotient and remainder. Then integrate quotient, integrate remainder, and add results {ratios of polynomials integration}.

sum of functions integration

To integrate sum of functions {sum of functions integration}: Integral of |u(x) + v(x)| * dx = integral of u(x) * dx + integral of v(x) * dx. Integral of |u(x) - v(x)| * dx = integral of u(x) * dx - integral of v(x) * dx.

MATH>Calculus>Integration>Methods

improper integral

Definite integrals {improper integral} can have infinity as limit or integrate over an open interval. To integrate improper integrals, take integral limit as variable approaches infinity. If limit is infinity, split domain at value zero, integrate over both intervals, and add results.

method of exhaustion

To find curved-figure areas and volumes, add many small discrete triangular or trapezoidal areas {method of exhaustion} {exhaustion method}.

method of indivisibles

Magnitudes can be infinite numbers of small units {indivisibles method} {method of indivisibles}. Cavalieri invented this calculus forerunner [1629].

Simpson rule

For degree-n functions, definite integral over (a,b) is ((b - a) / (3*n)) * (sum from k = 0 to k = n of c * f(a + k * (b - a) / n)), where n is even integer, c = 4 if k is odd, c = 2 if k is even, and c = 1 if k = 0 {Simpson's rule} {Simpson rule}. Error is less than or equal to M * (b - a)^5 / (180 * n^4), where M is less than or equal to fourth-derivative absolute value.

Wallis formula

For functions sin^m(x), cos^m(x), and cos^m(x) * sin^m(x), where m is positive integer, such as sin(x) and sin^2(x), reduction formulas {Wallis's formula} {Wallis formula} can evaluate definite integrals from x = 0 to x = pi/2.

MATH>Calculus>Integration>Methods>Parts

integration by parts

Integral of u*dv equals u*v minus integral of v*du {parts integration} {integration by parts}.

composite function

For two functions that depend on the same variable {composite function integration}, integral of u(x) * dv(x) = u(x) * v(x) - integral of v(x) * du(x).

MATH>Calculus>Calculus Of Variations

calculus of variations

Calculus {calculus of variations} studies function maxima and minima (Euler and Lagrange).

optimum

Maximum or minimum is where function has derivative equal zero: f'(y) = 0. For f'(y) = 0 = df(y) / dx, integrating gives f(y) - f(y', x) - f(y', y) * y' - f(y', y') * y'' = 0, where x = maximum or minimum domain value, y' = dy / dx, and y'' = d^2y / dx^2, with d^2 for second derivative.

If function is continuous in closed bounded domain, it has maximum and minimum.

motion through fluid

Motion through fluid takes path with least action, such as least-resistance path, brachistochrone, geodesic, and isoperimetrical curves.

iteration

Calculus of variations solves problems involving friction, inhomogeneity, and anisotropy by iterative integration. First, transform integral into sum of terms, to change boundary-value problem into initial-value problem. Then use initial value to optimize variable, using envelope of curve tangents, not point set. Repeat using previous state, to re-optimize. Solution is typically disturbance that moves through space or time.

geodesic of space

Curved space has minimum path length {geodesic, calculus}|.

isoperimetrical curve

Curves {isoperimetrical curve} can bound maximum area.

MATH>Calculus>Differential Equation

differential equation

Equations {differential equation} can use derivatives to model change, without considering initial or boundary conditions.

purpose

Differential equations solve elasticity, spring, vibration, catenary, pendulum, astronomy, Earth-shape, and tractrix problems.

types

Ordinary differential equations have no partial derivatives. Partial differential equations have at least one partial derivative.

order

Differential equation has highest derivative {order, differential}.

degree

Differential equations have variable power {degree, equation} in highest-order derivative. First-degree differential equations {linear differential equation, degree} model additive phenomena.

ordinary differential equation

Differential equations {ordinary differential equation} with no partial derivatives can model past or future conditions over time.

MATH>Calculus>Differential Equation>Methods

general solution of differentiation

Integrating differential equation removes derivative and finds solution {general solution, differentiation}. Equations with nth derivative integrate n times. Variable power in highest-order derivative determines integration method.

integration constant

Because derivatives of constants are zero, general solutions are true to within an additive constant. General solutions are solution envelopes.

initial condition

Knowing one function value {boundary value} {initial value} or function derivative {initial condition} allows finding integration constant and so exact solution {particular solution} {singular solution}. If equation has nth derivative, n initial conditions find exact solution. Sum of general solution and particular solution is solution {complete solution}.

partial differential equations

Using variable-separation methods and/or infinite-series methods, to make ordinary differential equations, can solve partial differential equations.

conditions

To model situations that depend on conditions, use same differential equation and add integral equation to account for conditions separately.

existence proofs

To prove solution existence, demonstrate condition {Lipschitz condition}, demonstrate theorem {Cauchy-Lipschitz theorem}, or use iteration to reach solution {successive approximation method} {method of successive approximation} {existence of solutions}.

company

Terms {company} on non-homogeneous differential-equation right side can be similar to terms on left side.

method of perturbations

Slight deviations from conic sections {method of perturbations} {perturbations method} can solve differential equations.

method of undetermined coefficients

To solve differential equations with derivatives of x^n, sin(a*x), or cos(a*x), reset coefficients to one or zero, solve, and then put back coefficients {method of undetermined coefficients} {undetermined coefficients method}.

method of variation of constants of integration

Small integral-value changes {method of variation of constants of integration} {variation of constants of integration method} can solve differential equations.

method of variation of parameters

To solve differential equations with derivatives of functions that are not x^n, sin(a*x), or cos(a*x), use parameters to make ordinary differential equation and vary parameters to simplify equation {method of variation of parameters} {parameter variation method} {variation of parameters method}.

power series method

To solve differential equations with derivatives of functions that are not x^n, sin(a*x), or cos(a*x), substitute power series, such as Taylor series, for function {power series method}.

relaxation method in mathematics

Substituting with algebraic equations can solve differential-equation systems {relaxation method, mathematics} {relaxation process}. Over an interval, select number of discrete points equal to number of variables in differential equations. At points, find approximate function values. At points, find partial derivative slope {differential coefficient} with respect to each variable: Df(x(i)) / Dx(i), where D denotes partial derivative, x is variable, and i is point/variable index. Write same number of algebraic equations as number of variables and points, each with a differential coefficient. Solve algebraic-equation system by computer.

iteration

Recognition algorithms can use iteration to move simultaneously toward optimum parameter values. Enhance some frequencies. Correlate with template. Equalize frequency histogram for more contrast. Subtract slowly varying information {background, recognition}. Find edge that has fast intensity change, using templates. Find surface orientations by neighboring reflectances. Find distances. Find velocities by comparing succeeding images. Find discontinuities and continuities.

MATH>Calculus>Differential Equation>Methods>Boundary

boundary value problem

After integration, solutions need point {boundary value, solution} {initial value, solution} to find integration constant. Problem can have no boundary or initial value {boundary value problem} {initial value problem}. Method of arithmetic means and method of sweeping out can find solutions to ordinary and partial differential-equation systems.

Dirichlet problem

Potential function or harmonic function may or may not exist at boundary {Dirichlet problem} {first boundary-value problem}.

MATH>Calculus>Differential Equation>Methods>Homogeneous

indicial equation

For homogeneous differential equations, equations {indicial equation} {characteristic equation, solution} can find solutions using base e raised to a power. r^n + a1 * r^(n - 1) + a2 * r^(n - 2) + ... + an = 0, where n is equation order, and r is general-solution highest power of e. Indicial equations remove highest-power term from differential equations, reducing equation degree.

integrating factor

Factors {integrating factor} can multiply an equation to make equation homogeneous.

method of separation of variables

To solve homogeneous differential equations, isolate variables {method of separation of variables} {separation of variables method}. Roots are e^(q*x) * (a + b*x + c * x^2 + ...), where q is coefficient, x is independent variable, and a b c are coefficients.

MATH>Calculus>Differential Equation>Methods>Partial

characteristics method

For first-order partial differential equations with n variables, variable separation can make ordinary differential equations with n parameters {method of characteristics} {characteristics method}. Characteristic curves and integrals are envelopes.

Lagrange method

For first-order partial differential equations, variable separation can result in ordinary differential equations with parameters {Lagrange method}.

majorant function

Power series with convergence domain can solve partial-differential-equation systems {method of majorant functions} {majorant function method}.

method of singularities

Green's theorem and Green's function can solve partial differential equations {method of singularities} {singularities method}.

MATH>Calculus>Differential Equation>Singularity

singularity in solution

Differential-equation solution can have infinite value {singularity, solution} at point {singular point, differential equation}. Singular point has at least one discontinuous differential coefficient. Singular point can be stable at focal point, where all curves through the point are convex. Singular point can be stable at center. Singular point can be unstable at point {node, intersection} where paths meet and end.

Fuchsian theory

Theories {Fuchsian theory} can smooth singularities in linear differential equations.

monodromy group

Groups {monodromy group} can explain singularities in linear differential equations.

saddle point

Singular points {saddle point}| can be unstable where convex and concave curves are orthogonal.

MATH>Calculus>Differential Equation>Kinds

adjoint equation

Non-homogeneous nth-order differential equations {adjoint equation} can have non-constant coefficients.

Bernoulli equation

dy/dx = p(x) * y + q(x) * y^a, where a is not zero and a is not one {Bernoulli equation}.

Clairaut equation

y = x * y' + f(y') {Clairaut's equation} {Clairaut equation}.

factorial equation

Y(n,z) = sum from n = 0 to n = infinity of ((-1)^r * (z/2)^(n + 2*r) / (r! * (n + r)!)) * (2 * log(z/2) + 2*c - (sum from m = 1 to m = n + r of (1/m)) - (sum from m = 1 to m = r of (1/m))) - (sum from r = 0 to r = n - 1 of (z/2)^(2*r - n) * (n - r - 1)! / r!) {factorial equation}.

Hermite function

Special functions {Hermite function} can solve ordinary differential equations over infinite or semi-infinite intervals.

Lame differential equation

(p^2 - h^2) * (r^2 - k^2) * ((d^2)E(r) / (dr)^2) + r * (2 * p^2 - h^2 - k^2) * (dE(r) / dr) + ((h^2 + k^2) * p - n * (n + 1) * r^2) * E(r) = 0, where (d^2) is second derivative, r is radius, p is vertical dimension, n is parameter, and (k,h) is point {Lamé's differential equation} {Lamé differential equation}. Solution functions {Lamé function} are elliptical harmonics of first or second kind.

linear differential equation

Linear equation has variables raised only to first power. Differential equation has derivatives {linear differential equation, calculus}. Second-order differential equation has second derivatives. Homogeneous equation has function equal to zero.

homogeneous

Second-order first-degree linear homogeneous differential general equation is a * (d^2)x + b * dx + c = 0, where (d^2) is second differential, d is first differential, x is independent variable, and a, b, and c are coefficients. General solution is c1 * e^(r1 * x) + c2 * e^(r2 * x), where e is base of natural logarithms, c1 and c2 are constants, r1 and r2 are roots, and x is independent variable.

non-homogeneous

Second-order, first-degree linear non-homogeneous differential general equation is a * (d^2)x + b * dx = -c, with same general solution.

multiple-valued function theory

Functions can have multiple values. Partial-differential-equation systems can model multiple-value functions {theory of multiple-valued functions} {multiple-valued function theory}.

non-linear differential equation

To solve non-linear differential equations {non-linear differential equation}, look for stable point using qualitative theory or find characteristic equation, using theorems {Poincaré-Bendixson theorem} and operations {Painleve transcendent operations}.

Riccati equation

dy/dx = a0(x) + a1(x) * y + a2(x) * y^2 {Riccati equation}.

Sturm-Liouville theory

Second-order ordinary differential equation can expand into infinite series of eigenfunctions {Sturm-Liouville theory, differential equation}.

three-body problem

No exact model exists for three mutually gravitationally interacting bodies {three-body problem}. Approximate solutions model mass-center straight-line motions and use energy and momentum conservation laws.

two dimensions

Perhaps, physical problems in three dimensions can reduce to problems in two dimensions using information concepts. Information is on surface, instead of in volume. Projection onto surface from volume has same information about positions, momenta, and transition probabilities. For special cases, reducing to two dimensions can solve the three-body problem.

Weber function

Functions {Mathieu function} {Weber function}, in mutually orthogonal curvilinear coordinates, can solve the potential equation.

MATH>Calculus>Differential Equation>Kinds>Complex

automorphic function

Circular, elliptic, hyperbolic, and other analytic functions {automorphic function} can generalize to find higher properties.

invariance

Automorphic functions are invariant if z' = (a*z + b) / (c*z + d) where a*d - b*c = 1, z is complex number, and z' is complex conjugate.

theta

theta(z) = sum from i = 0 to i = infinity of (c(i) * z + d(i))^(-2 * m) * H(z(i)), where m > 1 and H is rational function. Automorphic-function groups can be discrete or discontinuous groups of infinite order {theory of automorphic functions, discontinuous} {automorphic function theory, discontinuous}.

Bessel equation

(d^2)u / (dr)^2 + du / (r * dr) + a^2 - b^2 / r^2 = 0 {Bessel equation}. x^2 * y" + x * y' + (x^2 - n^2) * y = 0, where (d^2) is second derivative and x and n are complex, has two solutions. J(n,x) = (1 / (2 * pi)) * (integral from u = 0 to u = 2*pi of (cos(n*u) - x * sin(u)) * du). x * J(n+1,x) - 2 * n * J(n,x) + x * J(n-1,x) = 0.

MATH>Calculus>Differential Equation>Kinds>Partial

partial differential equation

Partial differential equation {partial differential equation} can have order greater than one, with second or higher derivatives. Partial differential equations of order greater than one are equivalent to first-order partial-differential-equation systems {system of partial differential equations}. For example, homogeneous, linear, second-order partial differential equation can be two first-order partial differential equations. c1 * (D^2)x + c2 * Dx + c3 = 0, where (D^2) is second derivative, D is first derivative, and c1, c2, and c3 are constants. c11 * Dx + c12 = 0 and d21 * Dx + d22 = 0, where D is first derivative and c11, c12, c21, and c22 are constants.

conditions

Partial differential equations can use boundary values and initial values.

method of arithmetic means

Methods {arithmetic means method} {method of arithmetic means} {sweeping out method} {method of sweeping out} similar to ordinary-differential-equation methods can find partial-differential equation-system solutions.

heat-flow equation

Partial differential equation {heat-flow equation} {heat equation} can represent heat flow. Second derivatives of heat with respect to distance equal constant squared times first partial derivative of heat with respect to time: (D^2)T / Dx + (D^2)T / Dy + (D^2)T / Dz = (k^2) * (DT / Dt), where T is heat, (D^2) is second partial derivative, D is partial derivative, k is constant, and x, y, z, and t are coordinates.

eigenfunction

Variable separation on partial differential equations can result in ordinary differential equations that use parameters {eigenfunction}| that have value sequences {eigenvalue, mathematics}. Ordinary differential equation solutions use eigenvalues. Second-order ordinary differential equations can expand into infinite series of eigenfunctions {Sturm-Liouville theory, eigenfunction}.

Euler theorem

For homogeneous functions u with n variables, n*u = x * (Du/Dx) + y * (Du/Dy) + ..., where D are partial differentials {Euler's theorem} {Euler theorem}.

Navier-Stokes equation

First-order partial differential equations {Navier-Stokes equation} describe fluid dynamics, using velocity, pressure, density, and viscosity. Examples are fluid motions and viscous-media object motions.

Plateau problem

Partial differential equations {Plateau's problem} {Plateau problem} can represent surfaces of least area under closed boundaries. Example is soap film in loop.

total differential

Partial differential equations {total differential equation} can be P*dx + Q*dy + R*dz = 0.

MATH>Calculus>Differential Equation>Kinds>Partial>Potential

excess function

Partial differential equations {excess function} {E-function} can represent energy function.

least constraint

Energy or force equations can minimize quantities {least constraint principle} {principle of least constraint}. For example, sum of kinetic-energy-to-potential-energy changes over time {action} can be minimum: integral of (kinetic energy - potential energy) * dt.

Hamilton-Jacobi equation

Partial differential equations {Hamilton-Jacobi equation} can represent potential energy plus kinetic energy equals total energy. Sum of second partial derivatives of potential with respect to each coordinate and partial derivative of potential with respect to time equals zero: (D^2)V / Dx + (D^2)V / Dy + (D^2)V / Dz - DV / Dt = 0, where V is potential, (D^2) is second partial derivative, D is partial derivative, and x, y, z, and t are coordinates.

Laplace operator

Operators {Laplace operator} {Laplace's operator}, on vector fields or potentials {del squared of f}, can be second derivatives, describe field-variation smoothness, be vectors, and be non-linear.

potential

Partial differential equations {potential equation} {Laplace's equation} can represent potentials. Potential V depends on distance r from mass or charge center: r = (x^2 + y^2 + z^2)^0.5.

Second partial derivative of potential V with respect to distance along x-axis plus second partial derivative of potential V with respect to distance along y-axis plus second partial derivative of potential V with respect to distance along z-axis equals zero: (D^2)V / Dx + (D^2)V / Dy + (D^2)V / Dz = 0, where (D^2) is second partial derivative, D is partial derivative, and V is constant times distance from center, because dx^2 / dx = 2 * x and d(2*x) / dx = 0.

solution

Spherical functions or Legendre polynomials can solve potential equation.

Legendre differential

(1 - x^2) * y'' - 2 * x * y' + n * (n + 1) * y = 0, where n is parameter {Legendre differential equation}. Solutions are polynomials {Legendre polynomial}, potential equation spherical coordinates derived by variable separation, or spherical harmonics of second kind.

Neumann problem

For boundaries with potential change zero, calculations can find potential change normal to region {Neumann problem} {second fundamental problem}.

Poisson equation

If potential-equation right side equals -4 * pi * (energy density), rather than zero, equation describes gravitation and electrostatics {Poisson's equation} {Poisson equation}. Energy density is pressure.

MATH>Calculus>Differential Equation>Kinds>Partial>Wave

periodic function

Functions {periodic function} can solve partial differential equations (D^2)y / Dt = (a^2) * ((D^2)y / Dx), where (D^2) is second partial derivative, D is partial derivative, a is constant, t is time, x is distance, and y is function of time and distance. Representing functions by infinite trigonometric series can solve periodic equations. Parameters can analyze function, so y(t,x) = h(t) * g(x). Parameters set equation eigenfunction and eigenvalues.

electromagnetic wave equation

First-order partial differential equation {electromagnetic wave equation} describes electromagnetic-wave energy oscillations.

cylindrical wave

Waves {cylindrical wave} can have partial differential equations. Second partial derivative of velocity with respect to time, times 1/c^2, equals three times partial derivative of velocity with respect to distance along pipe length, times 1/z, plus second partial derivative of velocity with respect to distance: ((D^2)v / Dt) * (1 / c^2) = 3 * (Dv / Dz) * (1/z) + (D^2)v / Dz, where (D^2) is second partial derivative, D is partial derivative, v is velocity, z is distance, t is time, and c is constant.

spherical wave

Waves {spherical wave} can have partial differential equations. Second partial derivative of radial velocity with respect to time, times 1/c^2, equals four times partial derivative of radial velocity with respect to radius, times 1/V, plus second partial derivative of radial velocity with respect to radius: ((D^2)s / Dt) * (1/c^2) = 4 * (Ds / DV) * (1/V) + (D^2)v / DV, where (D^2) is second partial derivative, D is partial derivative, v is radial velocity (ds/dt), c is constant, and radius V = (x^2 + y^2 + z^2)^0.5.

stationary wave equation

Vibrators with fixed endpoints can have stationary waves. Wave equations {stationary wave equation} can model steady-state waves. Wavefunction del operator, potential energy change, plus constant times wavefunction, kinetic energy change, equals zero {reduced wave equation} {Helmholtz equation}: Dw + (k^2) * w = 0, where w is wavefunction, D is delta function, and k is constant. The solution is an exponential function with complex exponents.

MATH>Calculus>Series

series

Terms can add or multiply in sequence {series, mathematics} {mathematical series}. Series has sum or product. Series has general term and follows rule. For example, 1 + 2 + 3 + ... is series that has general term n and follows the rule that next term is one higher than previous term. Series sum through nth position is n * (n + 1) / 2.

recursion formula

For series, a formula {recursion formula} gives next term from previous term.

partial sum

For series, sum {partial sum} adds first term through nth term: Sn = sum from k = 1 to k = n of general term a(k). Other sums {averaging partial sums} {Holder summability} {Cesaro sum} include converting divergent series to continued fractions, or vice versa.

partial product

For series, a product {partial product} multiplies first term through nth term: Pn = product from k = 1 to k = n of general term a(k).

error term

Power series can have error term {error term} {remainder, series}, which can have Taylor error term form and other forms {Schlömilch form} {Lagrange form} {Cauchy form}.

MATH>Calculus>Series>Sequence

sequence

Series {sequence, mathematics} {mathematical sequence} can have numbers or terms {ordered term} in sequence. Sequence has general term {general term, sequence} and follows rule. Example sequence is x, 2*x, 3*x, ... General term a(n) = n * x, where n is term position. The rule is that n increases by one.

separator

Commas separate sequence terms, as in 1, 2, 3, ...

types

Sequences can descend, ascend, or alternate. Sequence terms can approach number {convergent sequence}. Sequence terms can approach infinity {divergent sequence}. Sequence can be neither convergent nor divergent {indefinite sequence}. Sequence can increase, decrease, increase, and so on {oscillating sequence} {alternating sequence}.

coincident sequence

Two sequences are equal {coincident sequence} if and only if all corresponding sequence terms are equal.

induction axiom

The only sequence whose first term equals zero, and whose n+1th term equals zero if nth term equals zero, is sequence of zeroes {induction axiom}. The only sequence whose first term equals one, and whose n+1th term equals k + 1 if nth term equals k, is the positive-integer sequence. The positive-integer series is the only sequence that has the number one and contains the positive integers.

MATH>Calculus>Series>Convergence

convergence of series

If successive-term absolute value is less than previous-term absolute value, series converges {absolutely convergent} {convergence, series}.

Series can be convergent even if successive-term absolute value is not less than previous-term absolute value {conditionally convergent}. Conditionally convergent series can rearrange to make sum be any number.

constant times sequence

If sequence converges, limit of constant times sequence is constant times sequence limit.

uniform

Absolute value of partial sum S(n) minus sum from x = 1 to x = n of S(n) * x can be less than small value, for all x {uniform convergence}.

Abel summability

Convergent-power-series partial-sum limit is partial sum, if convergence radius replaces general-term independent variable {Abel summability}.

asymptotic series

Divergent series {asymptotic series} can represent functions and evaluate integrals. In such divergent series, nth-term error is less than n+1th-term absolute value, so error becomes less as term number increases.

bounded sequence

Absolute value of each sequence term can be less than or equal to a constant {bounded sequence}.

divergence of series

If ratio between next-term absolute value and previous-term absolute value is greater than or equal to one, sequence diverges {divergence, series}.

semiconvergence

Divergent series {semiconvergent series} can have nth-term error less than n+1th-term absolute value, so error decreases faster than terms increase. Though they diverge, semiconvergent series can evaluate integrals, because sum is finite. Useful asymptotic series is f(x) = a0 + a1/x + a2/(x^2) + ... If x is large, limit of x^n * (f(x) - series) = 0. x approaches 0 if 1/(x^n) * (f(x) - series) = a(n). Other ideas about asymptotic series include {Birkhoff's theorem} {Birkhoff's connection formula} {WKBJ solution} {Airy's integral}.

limit

If sequence successive term is less than previous term, and if general term is always less than constant, sequence has limit {limit, sequence} {sequence limit} {limit, series} less than or equal to constant.

Tauberian theorem

Summable series can make convergent series {Tauberian theorem}.

MATH>Calculus>Series>Convergence>Radius

radius of convergence

Independent variable can have value {radius of convergence} {convergence radius} greater than zero at which power series changes from convergence to divergence. Power series, power-series differential, and power-series integral have same convergence radius.

circle of convergence

For complex-number power series, if complex number lies within a complex-plane circle {convergence circle} {circle of convergence} centered on zero, with no singularities, series converges. If complex number lies outside a complex-plane circle, series diverges.

annulus of convergence

Laurent series has complex number that lies within annulus in complex plane {convergence annulus} {annulus of convergence}.

convergence region

Independent variable can have values {region of convergence} {convergence region, series} for which series converges.

MATH>Calculus>Series>Convergence>Test

Cauchy convergence criterion

If and only if absolute value of difference between successive partial sums is less than small value {Cauchy convergence criterion}, series converges.

comparison test

Sequence general term can be less than or equal to constant times second-sequence general term {comparison test}. If second sequence diverges, first sequence diverges. If second sequence converges, first sequence converges.

If second sequence converges, and if second-sequence general term divided by sequence general term has limit, first sequence converges.

If second sequence diverges, and second-sequence general term divided by sequence general term has limit or if quotient is infinite, first sequence diverges.

If limit of quotient of sequence general terms does not equal zero, both sequences either diverge or converge.

Dirichlet integral

Integral from x = 0 to x = a of (f(x) * sin(u*x) / sin(x)) * dx, and integral from x = a to x = b of (f(x) * sin(u*x) / sin(x)) * dx {Dirichlet integral}, where b > a > 0, can show convergence.

integral test

Sequences converge if and only if integral of general term, from x equals some value to x equals infinity, exists {integral test}.

Leibniz test

Alternating sequences can converge {Leibniz's test} {Leibniz test}.

ratio test

If successive-term to previous-term ratio limit is less than one, sequence converges {ratio test}. If successive-term to previous-term ratio limit is greater than one, sequence diverges. If successive-term to previous-term ratio limit is one, sequence can converge or diverge. If general-term limit equals zero, successive-term to previous-term-ratio absolute-value limit is less than one. Generalized ratio test {d'Alembert's test} exists.

MATH>Calculus>Series>Mean

arithmetic mean

To average sequence terms {arithmetic mean}, add all terms and divide by number of terms: (sum from k = 1 to k = n of a(k)) / n, where a(k) is general term, k is sequence position, and n is number of terms.

geometric mean

To average sequence terms {geometric mean}|, multiply all sequence numbers to get product, and then take product term-number root: (product from k = 1 to k = n of a(k))^(1/n), where a(k) is general term, k is sequence position, and n is term number.

harmonic mean

To average sequence terms {harmonic mean}|, divide number of terms by sum of sequence-term reciprocals: n / (sum from k = 1 to k = n of 1/a(k)), where a(k) is general term, k is sequence position, and n is number of terms.

Cauchy principle

Arithmetic mean {mean, series} can be greater than or equal to geometric mean, which is greater than or equal to harmonic mean {Cauchy's principle} {Cauchy principle}.

MATH>Calculus>Series>Kinds

convergent polynomial series

Continuous functions over real-number closed intervals can be absolutely and uniformly convergent series of polynomials {convergent polynomial series}.

monotone series

Series {monotone series} can be single-valued, have bounds, be piecewise continuous, and have finite numbers of discontinuities, maxima, and minima.

null sequence

Sequences {null sequence} can have zero as limit. Sequence-element absolute values can be less than positive rational numbers.

ordered sequence

Sequences {ordered sequence} can have one-to-one correspondence with positive integers.

MATH>Calculus>Series>Kinds>Multiple

sum of sequences

Sequences {sum of sequences} can have terms that are sums of terms of two other sequences. For sequences that converge, limit of sum sequence is sum of original-sequence limits.

product sequence

Sequences {product sequence} can have terms that are products of terms of two other sequences. For sequences that converge, limit of product sequence is product of original sequence limits.

quotient sequence

Sequences {quotient sequence} can have terms that are quotients of terms of two other sequences. For sequences that converge, limit of quotient sequence is quotient of original sequence limits.

MATH>Calculus>Series>Kinds>Number

Fermat numbers series

Number series can have general term 2^(2^n) + 1, for n = 0, 1, 2, ... {Fermat's numbers} {Fermat numbers}.

Fibonacci ratio

Fibonacci sequences have consecutive-number ratios that approach golden section, (1 + 5^0.5) / 2 {Fibonacci ratio}.

Gregory series

pi = 4 * (1 - 1/3 + 1/5 - 1/7 + 1/9 - ...) {Gregory's series} {Gregory series}.

polygonal number

Arithmetic progression can have first term one and common difference n - 2, where n is number of polygon sides {polygonal number}.

n can be three, so sequence is 1, 3, 6, 10, 15, 21, ... {triangular number}.

n can be four, so sequence is 1, 4, 9, 16, 25, ... {square number, polygonal}.

n can be five, so sequence is 1, 5, 12, 22, ... {pentagonal number}.

For n, sequence is l, n, 3*n - 3, 6*n - 8, ... {n-gonal number}.

general

In general, 0.5 * (r + 1) * (r*n - 2*r + 2), where r is whole number, makes polygonal numbers.

MATH>Calculus>Series>Kinds>Progression

arithmetic progression

Difference {common difference} between consecutive sequence terms can be constant {arithmetic progression}. General term is a(n) = a(1) + d * (n - 1), where d is common difference. First-n-terms partial sum is n times average of first and last terms: Sn = n * (a(1) + a(n)) / 2, where a(k) is general term, and n is number of terms.

geometric progression

Consecutive sequence-term ratios {common ratio} can be constant {geometric progression}. General term is a(n) = a(1) * r^(n - 1), where r is common ratio, and n is number of terms. Partial sum is Sn = a(1) * (1 - r^n) / (1 - r), where r is common ratio, and n is number of terms.

harmonic progression

Difference between reciprocals of successive sequence terms can be constant {harmonic progression}. General term a(n) is 1 / a(n) = 1 / (a(1) + d * (n - 1)), where d is difference.

MATH>Calculus>Series>Kinds>Power

power series

Sequence general term can be a(n) * x^n {power series}|, where a are rational coefficients, and n is number of terms. Power series converges if independent variable equals zero. x^n / n! converges at all x.

associated series

1 + a1 * u + a2 * u^2 / 2! + ... {associated series}, where ai are series coefficients.

expansion of function

Power series {expansion, function} {expansion, series} can replace function or integral.

exponential series

e^x = 1 + x + x^2 / 2! + x^3 / 3! + ... {exponential series}.

hyperbolic expansion

sinh(x) = x + x^3 / 3! + x^5 / 5! + ... {hyperbolic expansion}. cosh(x) = 1 + x^2 / 2! + x^4 / 4! + ...

hypergeometric series

1 + a*b*x / c + (a * (a + 1) * b * (b + 1) * x^2) / (2 * c * (c + 1)) + ... {hypergeometric series} can converge at x. If x = 1, function equals (gamma(c) * gamma(c - a - b)) / (gamma(c - a) * gamma(c - b)).

Laguerre series

1 + x + 2! * x^2 + 3! * x^3 + ... {Laguerre series}.

logarithmic series

ln(1 + x) = x - x^2 / 2 + x^3 / 3 - x^4 / 4 + ... {logarithmic series}, where -1 <= x <+ +1. ln(N + 1) - ln(N) = 2 * (1 / (2*N - 1) + 1 / (3 * (2*N - 1)^3) + 1 / (5 * (2*N - 1)^5) + ...).

Maclaurin expansion

f(b) = f(0) + (sum from i = 0 to i = n - 1 of (function ith derivative at 0) / i!) * b^i + error term {Maclaurin expansion}. Maclaurin expansion is Taylor series with a = 0.

radix fraction

Fractions {radix fraction} can be sums of positive common fractions a/r + b / r^2 + c / r^3 + ..., where r is a number, and a b c are integers. Common fractions can be radix fractions, using any radix in any number system.

Taylor theorem

f(b) = f(a) + (sum from i = 0 to i = n - 1 of (function ith derivative at a) / i!) * (b - a)^i + error term {Taylor's theorem} {Taylor theorem} {Taylor series}. For two variables, Taylor series has twice as many terms {Taylor expansion}.

MATH>Calculus>Series>Kinds>Trigonometric

trigonometric series

Over intervals, periodic functions can be infinite sine and cosine series {trigonometric series}.

Fourier analysis

Trigonometric series {Fourier series} {Fourier integral} can represent function over interval: (1 / pi) * (integral from -a = infinity to a = +infinity of F(a) * da) * (integral from a = 0 to a = infinity of cos(q * (x - a)) * da). Complex waveforms over time or position can be finite or infinite series of harmonic sine and cosine waves {Fourier analysis}|: f(x) = (2 * pi)^-0.5 * (integral from -infinity to +infinity of g(p) * e^(i*x*p) * dp), where g(p) is density {Fourier transform, series}. Complex function can have g(p) = 0 for p >= 0 {positive frequency function}.

convergence

If Fourier series is single-valued, has a bound, is piecewise continuous, and has finite numbers of discontinuities, maxima, and minima {Dirichlet condition}, it converges.

domain

Domain can be circle whose circumference is period or wavelength.

range

Series can be on unit circle in complex plane.

theorem

Fourier-series coefficients can exist and have properties {Parseval's theorem}.

Laurent series

Sines and cosines of Fourier series can be complex exponentials {Laurent series}: F(z) = F(e^i * a * x) = sum from - infinity to + infinity of Ar * z^r, where z is complex number, Ar is general term, and r is convergence radius. Laurent series has convergence annulus on Riemann sphere. Convergence circle can be for positive-term sum {positive frequency part}. Convergence circle can be for negative-term sum {negative frequency part}.

MATH>Calculus>Vector

vector in mathematics

Line segment {vector, mathematics} can have magnitude and direction.

length

Vectors have length {modulus, vector} {absolute value, vector}. Vectors {unit vector} can have length equal to one unit.

direction

Vectors have angle to x-axis {amplitude, vector}.

equality

Equal vectors have same sense, direction, and length but do not have to have same space position.

addition

To add two vectors, move vector {component} so initial end coincides with other-vector terminal end. Keep component vector parallel to original direction. Sum is vector resultant from first-vector initial end to component-vector terminal end.

To add two vectors, add corresponding coordinates to get resultant-vector coordinates: (1,2) + (3,4) = (4,6).

Vector addition is commutative and associative.

subtraction

To subtract vectors, first reverse direction of vector to subtract. Then add the vectors.

To subtract vectors, subtract corresponding coordinates: (1,2) - (3,4) = (-2,-2).

vector function

Ordered pairs can have real numbers as domain and vectors as range.

bundle

Vectors can have initial ends at same point {bound vectors} {bundle, vector} {vector, bundle}.

triangle of vectors

Two component vectors and their resultant make triangle {triangle of vectors}.

tensor

Vectors are rank-1 covariant tensors.

vector types

Vectors can have structures at termini. Structure can be another vector, such as a rotating vector. Structure can be a differential length, area, or volume.

funicular polygon

Coplanar vectors can make polygons {funicular polygon}. Coplanar-vector sum makes one polygon side.

Lame theorem

If three vectors have resultant zero, then vectors are in same plane, and vectors are proportional to sine of angle between other two vectors {Lamé's theorem} {Lamé theorem}.

version of vector

Tie cube by eight threads, one from each cube corner to each room corner {orientation-entanglement relation}. Rotate cube {version, vector}| 360 degrees. Orientation is same as before, but threads no longer go straight to corners. Rotate cube total of 720 degrees, so orientation is same as before. Now threads go straight to corners.

MATH>Calculus>Vector>Properties

magnitude of vector

Vector has absolute value {length, vector} {magnitude, vector}|.

direction angle

Vectors make angles {direction angle} with coordinate axes. Direction angles have cosines {direction cosine}. Space curves have derivatives of tangent, binormal, and normal with respect to direction cosines {Serret-Frenet formula}.

direction number

Vectors have coordinate values {direction number} along dimensions.

MATH>Calculus>Vector>Field

vector field in mathematics

Spaces can be continuous point, vector, loop, string, spinor, or other-object sets. Fields {vector field, mathematics} can have vectors at all points. Force and momentum fields are vector fields. Scalar-field gradients are vector fields, because gradients have direction and magnitude.

scalar field in mathematics

Fields {scalar field, mathematics} can have magnitude, like temperature or position, at points.

MATH>Calculus>Vector>Operations

couple

For mappings, sets of joins {couple, vector} are correspondences.

curl operation

Distance-vector-field curls {curl, operation} are vector fields. If distance vector fields make 2x2 determinants, curls calculate area: a X b. If distance vector fields make 3x3 determinants, curls calculate volume: a . (b X c). If distance vector fields make 4x4 determinants, curls calculate four-volume: (a X b) . (c X d).

flux of vector

Force-vector-field divergence makes a scalar field {flux, vector}| indicating total lines through a point or surface.

parallel transport

Move vector on geodesic while keeping vector tangent to geodesic and parallel to original vector direction, so gradient along vector equals zero {parallel displacement} {parallel transfer} {parallel transport}|.

projection of vector

Vectors can project onto line using perpendiculars from vector endpoints to line {projection, vector}.

rotation operator

Vectors can rotate around {rotation operator} own axis, coordinate axis, or any axis. Vectors can rotate around points.

rotation only

Complex rotation operator is 1 - (i/2) * (Ax * dt(y, z) + Ay * dt(z, x) + Az * dt(x, y)), where t is time and Ax = |0, 1, 1, 0|, Ay = |0, -1, i, 0|, and Az = |1, 0, 0, -1|. A^2 = 1. Ax * Ay = - Ay * Ax = i * Az. Operator changes vector direction but not length or origin {active transformation}.

Product of two rotation operators gives result of two rotations. Rotations have inverses. Rotation and inverse rotation result in no rotation, so product is zero.

translation

Complex translation operators can changes reference frame, but vector maintains direction and length {passive transformation}. cos(t/2) - (i * sin(t/2)) * (Ax * cos(a) + Ay * cos(b) + Az * cos(c)), where t is rotation angle, Ax = |0, 1, 1, 0|, Ay = |0, -1, i, 0|, and Az = |1, 0, 0, -1|. A^2 = 1. Ax * Ay = - Ay * Ax = i * Az. i^2 = j^2 = k^2 = i * j * k = 1. j * k = 1. k * j = -1. k * i = j. i * k = -j. i * j = k. j * i = - k.

reflection

If rotation angle is t, rotation is equivalent to two successive reflections in two planes that meet at angle t/2.

MATH>Calculus>Vector>Operations>Sum

resolution of vectors

Two component vectors can add to result in one resultant vector {resolution, vector}|.

resultant vector

Vector sum {resultant vector}| is vector from first-vector initial end to component-vector terminal end.

MATH>Calculus>Vector>Operations>Product

bivector

Vectors are directed line segments. Two vectors with same origin {bivector} represent a directed plane segment. Bivector attitude, orientation, rotation, or gradient is direction in space of plane-segment normal compared to coordinate axes. Bivector direction, rotation sense, or circulation is sense of rotation of first vector into second vector (clockwise or counterclockwise). Bivector symbol is /| {wedge symbol} between two vectors.

The wedge product of two vectors makes a bivector. Wedge product squared equals negative of first-vector magnitude squared times second-vector magnitude squared times sine of angle A between vectors squared: (a /| b)^2 = -|a|^2 * |b|^2 * sin^2(A). Therefore, bivectors are imaginary numbers, and bivectors are not vectors or scalars.

Bivector magnitude is the parallelogram area: |a| * |b| * sin(A). If bivector magnitude equals zero, vectors are parallel, collinear, or linearly dependent.

Three vectors with same origin {trivector} represent a directed volume segment. Trivector symbol is wedges between vectors: a /| b /| c. k vectors with same origin {k-vector} represent a directed k-volume segment. k vectors can combine to represent a directed k-volume segment. k-vectors can have parallel and perpendicular components.

Cross products of two vectors make a third vector perpendicular to both vectors. Wedge (exterior) products express cross products using only components in the plane made by the two vectors. For two vectors a = x1 * i + x2 * j and b = y1 * i + y2 * j, cross product is (x1*y2 - x2*y1) * k, and wedge product is (x1*y2 - x2*y1) times the ij bivector: (x1*y2 - x2*y1) * i/|j.

Two dimensions have one basis bivector: e12. Three dimensions have three basis bivectors: e23 = i, e31 = j, e12 = k. Three-dimensional bivectors have form a * e23 + b * e31 + c * e12.

Geometric product of a vector and a bivector is interior-product vector plus exterior-product trivector. (Commutator product is zero.) In two-dimensional space, this geometric product rotates the vector. In three-dimensional space, if, for example, vector is a1 * i and bivector is a2 * e23 + b2 * e31 + c2 * e12, geometric product is a1 * i * a2 * e23 + a1 * i * b2 * e31 + a1 * i * c2 * e12 = - a1 * a2 * e123 + + a1 * b2 * j - a1 * c2 * k.

Geometric product of two bivectors is interior-product scalar plus commutator-product bivector plus exterior-product quadrivector. Three-dimensional spaces have no exterior-product quadrivector. For example, if first bivector is a1 * e23 + b1 * e31 + c1 * e12 and second bivector is a2 * e23 + b2 * e31 + c2 * e12, geometric product is - a1 * a2 - b1 * b2 - c1 * c2 + (c1 * b2 - b1 * c2) * e23 + (a1 * c3 - c1 * a3) * e31 + (b1 * a2 - a1 * b2) * e12. Commutator product makes a plane segment. If first bivector is a2 * e23 and second bivector is a2 * e23, geometric product is 0. If first bivector is a1 * e23 and second bivector is b2 * e31, geometric product is a1 * b2 * e23 * e31 = - a1 * b2 * e12.

Cartesan product

Ordered pairs {Cartesan product} relate first member to second member. Cartesan product is correspondence. Inverse of Cartesan product has bold divide sign.

cross product

Vector products {cross product}| {vector product} {outer product} can result in vectors. Cross-product symbol is X: u X v.

magnitude

Cross-product-vector magnitude equals first-vector u length absolute value times second-vector v length absolute value times sine of angle A between vectors: |u| * |v| * sin(A).

direction

Cross-product-vector direction is perpendicular to both original vectors. Cross-product-vector sense is thumb direction if right-hand fingers curl in direction of positive angle between vectors.

coordinates

i-direction coordinate equals first-vector second coordinate x2 times second-vector third coordinate y3 minus first-vector third coordinate x3 times second-vector second coordinate y2. j-direction coordinate equals first-vector third coordinate x3 times second-vector first coordinate y1 minus first-vector first coordinate x1 times second-vector third coordinate y3. k-direction coordinate equals first-vector first coordinate x1 times second-vector second coordinate y2 minus first-vector second coordinate x2 times second-vector first coordinate y1. Therefore, cross-product vector is (x2 * y3 - x3 * y2) * i + (x3 * y1 - x1 * y3) * j + (x1 * y2 - x2 * y1) * k.

Unit-vector cross products make unit vectors. j X k = i. k X i = j. i X j = k. Unit-vector cross products with themselves equal zero: i X i = j X j = k X k = 0.

properties

Cross products are not commutative, because i X j = +k and j X i = -k. i X j = -j X i. i X k = -k X i. j X k = -k X j. Cross products are distributive: c * (i X j) = (c * i) X (c * j) = c * k. Cross products have no inverse, because there is no cross division. Cross products find forces and torques and so curve the function.

exterior product

Vector products {exterior product} {wedge product} can be cross products of two vectors expressed without using components outside vector plane. If a = x1 * i + x2 * j and b = y1 * i + y2 * j, wedge product is (x1 * y2 - x2 * y1) times bivector of i and j.

geometric product

For two geometric objects, sum of dot product and wedge product is a product {geometric product} (Clifford). Dot product makes geometric object one grade lower. Wedge product makes geometric object one grade higher.

grade

Wedge products have numbers {grade, vector}| of elements.

Grassmann product

Two operations make wedge products {Grassmann product, wedge product}.

right hand rule for vectors

For spread right hand, if straight fingers point in same direction as first vector and bent fingers point in same direction as second vector, vector product direction is thumb direction {right hand rule, vector}|. Positive angle is between first and second vector.

multivector

Geometric products sum vectors of different dimensions to make a vector type {multivector}.

scalar multiplication

Vector multiplied by constant {scalar multiplication} makes vector shorter or longer and changes modulus but does not change orientation. Multiplying vector by negative number changes sense, with same orientation but opposite direction. Vector coordinates are distributive over scalar multiplication.

scalar product

Products {scalar product}| {dot product} {inner product} of two vectors can result in scalars. Scalar product sign is bold dot: u . v, where u and v are vectors.

Scalar equals first vector-length u times second-vector length v times cosine of angle A between vectors: |u| * |v| * cos(A).

Scalar equals first-vector first coordinate x1 times first-vector second coordinate x2 plus second-vector first coordinate y1 times second-vector second coordinate y2: x1 * x2 + y1 * y2.

Both vectors can be the same: x*x + y*y = x^2 + y^2. Two vectors are parallel if they are scalar multiples. Two vectors are perpendicular if their scalar product equals zero.

Scalar product is commutative, is distributive, and has no inverse. Scalar products find energies and so where functions begin or end (boundaries).

MATH>Calculus>Vector>Tensor

tensor

Linear forms {tensor, mathematics} have dimension or variable coefficients. Scalars, vectors, and matrices are tensors. Vectors are rank-1 covariant tensors.

purposes

Tensors can transform coordinates. Tensors can sum over all component combinations or any component. Tensors describe vector operations, complex numbers, analytic geometry, and differential geometry. All physical laws are tensor relations. For example, tensors describe flow, crystal deformations, and elasticity. All intensive physical quantities can be tensors. Tensors can measure extensive quantities, such as mass, momentum, energy, inertia moment, length, area, and volume.

differentiation

To differentiate tensor, raise tensor order by one. Differentiating first-order tensor results in second-order tensor. Differentiating tensor makes tensor gradient.

integration

To integrate tensor, lower tensor order by one, by summing over one component.

area

Tensor transformations find surface areas, which are outer products. Tensor determinants give areas. Transformations can be second-order skew-symmetrical covariant tensors or skew-symmetric bilinear forms: sum over all ij of g(ij) * du *dv. Terms with same index, such as ii, have coefficient zero. Terms with different indexes, such as ij, have coefficient one. In covariant transformations, new coefficients are new-vector coefficients, and variable number stays the same.

invariants

Tensor invariants are distance, curvature, sum of curve partial-derivative squares, sum of curve second derivatives, sum of area second derivatives, and sum of volume second derivatives. Tensor invariants have both contravariant and covariant components. They can contract.

tensor density

Tensors can multiply metric-coefficient-determinant square roots. Tensor densities are contravariant and symmetric, like divergence. If vectors have same direction, metric-coefficient-determinant square root is zero.

polynomials

Many-variable polynomials can be equivalent to tensors {polynomial, tensor} {tensor, polynomial}. For example, double sums with two-variable terms have quadratic form a*x*x + b*x*y + c*y*y, which can be equivalent to scalar products. Differential forms can use dx instead of x.

order of tensor

Temperature and other scalars are quantities without direction, have no components, and are zero-order tensors {order, tensor}. Motion, momentum, force, and other vectors have one direction and are first-order tensors. Metric and other matrices can represent two-dimension interactions and are second-order tensors. Vector curvatures can represent four-dimension interactions and are fourth-order tensors.

MATH>Calculus>Vector>Tensor>Operations

Bianchi symmetry

Curvature tensor is 0 if there is no torsion {first Bianchi identity, symmetry} {Bianchi symmetry, tensor}.

Bianchi identity

Curvature-tensor derivative is 0 if there is no torsion {Bianchi identity, tensor} {second Bianchi identity, tensor}.

contravariant

Coordinates can depend directly on dimensions {contravariant, tensor}. Coefficients a times basis vectors e result in coordinates x: a(i) * e(i) = x(i).

covariant

Covariant components relate to contravariant components. If basis vectors are orthogonal, covariant components and contravariant components are equal: a(i) = x(i) / e(i).

If basis vectors are curved coordinates, then a(i) = g(i,j) * a(j), where g(i,j) depend on basis vectors e(i) and e(j). Some g(i,j) components are for covariance, some for contravariance, and some for both. g(i,j) tensor relates basis vectors. g(i,j) elements are functions of curved-space positions.

Euclidean space

g(i,j) elements are 1 or 0 for flat space with orthogonal basis vectors.

coordinate transformation

Physical quantities or coordinates can transform from one coordinate system to another {coordinate transformation}. First coordinate-system vector components are linear functions of second coordinate-system components.

tensor

Tensor coefficients are weights by which to multiply old variables to get new variables. Tensor-term number is old-component number times new-component number. Scalar product of outer-product tensor and old basis vectors obtains new basis-vector scalars.

projection

Linear transformation projects old onto new. Linear transformation is affine geometry.

contravariant

Contravariant component {contravariant}, such as dx, multiplies with tensor. Terms with same index, such as ii, have coefficient one. Terms with different indexes, such as ij, have coefficient zero. In contravariant transformation, only diagonal terms remain. Contravariant component sum is vector expressed in old basis vectors. Diagonal terms are scalars for new basis vectors.

covariant

Covariant component {covariant}, such as partial derivative, is contravariant component times tensor. Terms with different indexes, such as ij, have coefficient one. Terms with same index, such as ii, have coefficient zero. In covariant transformation, diagonal terms are not present. Covariant component sum is weight matrix. Non-diagonal terms are weights.

covariance and contravariance

Covariant means that different old and new components interact. Contravariant means that same old and new components interact. Together, they account for all interactions. Contravariant reduces dimension by one. Covariant does not change dimensions. If components are orthogonal, as in Euclidean space, covariant and contravariant components are the same. Contravariant or covariant transformation does not change symmetrical-tensor value. Contravariant or covariant transformation only changes sign of odd number of skew-symmetrical tensor transformations.

covariant

Linear functions can find coefficients of scalar products from original variables and basis vectors {covariant, tensor}: a(i) = x(i) * e(i). Covariant components relate to contravariant components by relations between basis vectors. If basis vectors are orthogonal, covariant components and contravariant components are equal. If basis vectors are curved coordinates, then a(i) = g(i,j) * a(j), where g(i,j) depend on basis vectors e(i) ... e(j). Some g(i,j) components are for covariance, some for contravariance, and some for both. g(i,j) tensor relates basis vectors. g(i,j) elements are functions of curved-space positions.

g(i,j) elements are 1 or 0 for flat space with orthogonal basis vectors.

covariant transformation

Terms with different indexes, such as ij, have coefficient one. In covariant transformation, new coefficients are new-vector coefficients, and variable number stays the same.

Einstein summation

Notation conventions {Einstein summation convention} can denote tensors.

projection by tensor

Coordinates have unit vectors, such as u for x-axis and v for y-axis. Vector from origin can have coordinates: (2,3) = 2*u + 3*v, where u and v are unit vectors for two-dimensions. Straight vector has constant slope.

Lines and surfaces can curve. At point, line or surface has curvature. Slope change indicates curvature amount. For two dimensions, two orthogonal directions, u and v, can change slope: du and dv, where d is differential. Total curvature has coefficients that depend on dimensions and is sum of changes along dimensions: (Df(u,v) / Dv) * du + (Df(u,v) / Du) * dv, where D is partial derivative and d is differential.

linear

Linear functions depend on one variable raised only to first power: C * x. Bilinear functions depend on product of first-power variables: C * x * y.

symmetry

Functions have symmetry if function variables can interchange, for example, they are Euclidean space dimensions.

tensor

Tensors are linear functions of coefficients times any number of variables: c * v1 * v2 * ... * vN, where c is coefficient, vi are variables, and N can be infinite. Tensors can be sums of these terms: c1 * i1 * j1 * ... * n1 + c2 * i2 * j2 * ... * n2 + ... + cN * iN * jN * ... * nN, where n can be infinite. Vectors are tensors: Cu * u or Cu * u + Cv * v.

scalar product

Bilinear forms are tensors: Cuv * du * dv. Bilinear tensors can be sum over all ij of g(ij) * du * dv, where i and j are dimensions, g is function of dimensions, and u and v are dimension unit vectors. Scalar product of two vectors (a,b) and (c,d) is a*c*i*i + b*d*j*j + a*d*i*j + b*c*j*i = a*c + b*d, which is symmetric bilinear tensor. In scalar products, terms with same index, such as ii, have coefficient one, and terms with different indexes, such as ij, have coefficient zero {contravariant transformation}. Vector projection onto itself makes 100% = 1 of vector. Vector projection onto perpendicular makes zero.

scalar product: symmetry

Tensor projection and scalar product are symmetric, because answer is the same if either vector projects onto other vector.

scalar product: projection

Scalar product projects one vector onto another to find length. Tensor transformations can project {projection, tensor} one vector onto another vector, to give length.

quadratic

If two vectors are the same, scalar product is quadratic. For vector (a,b), sum over all ij of g(ij) * du * du = a*a*i*i + b*b*j*j = a^2 + b^2.

cross product

Vector cross products are vectors and tensors: (a,b) and (c,d) make a*c*i*i + b*d*j*j + a*d*i*j + b*c*j*i = (a*d - b*d)*k, where j*i = - i*j = -k, because opposite direction. In cross products, terms with same index, such as ii, have coefficient zero, and terms with different indexes, such as ij, have coefficient one {covariant transformation}. Divergence of vector from itself is zero. Divergence of vector onto perpendicular makes 100% = 1 divergence. Tensors can be scalar, vector, matrix, and tensor products.

tensor contraction

Tensor order can reduce by two {tensor contraction}. If tensor has contravariant component and covariant component, sum tensor over each component, at same time, to eliminate each component.

trace of tensor

Scalar products contract second-order tensors and are sums of squares along diagonal {trace, tensor} {spur, tensor}.

volume using tensors

Volumes {volume, tensor} are determinants of third-order skew-symmetrical covariant tensors.

MATH>Calculus>Vector>Tensor>Kinds

basis vector

Space has dimensions. Vectors {basis vector} can lie along coordinate axes. Other vectors are basis-vector linear combinations.

contravariant

For point, coefficients a(i) times basis vectors e(i) result in point coordinates x(i): a(i) * e(i) = x(i), where i is number of dimensions. Points can move. New coefficients x(i) of same dimensions e(i) relate to old coefficients a(i): a(i) * e(i) = x(i).

covariant

For points, coordinate system can change. New-dimension coefficients a(i) relate to old dimensions e(i), because new dimensions are linear transformations x(i) of old dimensions: a(i) = x(i) * e(i).

relation

Covariant components relate to contravariant components. If basis vectors are orthogonal, covariant components and contravariant components are equal. If basis vectors are curved coordinates, a(i) = g(i,j) * a(j), where a(i) and a(j) are coefficients and g(i,j) is tensor relating basis vectors e(i) ... e(j). Some g(i,j) components are for covariance, some for contravariance, and some for both. g(i,j) elements are functions of curved-space positions. g(i,j) elements are 1 or 0 for flat space with orthogonal basis vectors.

coordinate system tensor

Tensor can express coordinates {coordinate system}. For example, space points can be vectors: a*i + b*j + c*k. Coordinates can be perpendicular or not perpendicular. Physical quantity depends on coordinates and can use tensor form. For example, momentum can be vector: mass * (a*i + b*j + c*k).

curvature of surface

Surface can deviate from flatness {curvature, tensor}.

local

Curvature is at point and is local property.

intrinsic

Curvature is intrinsic to surface and does not depend on outside reference points.

curve

For curves, curvature at point is curvature-radius reciprocal.

surface

For surface points, curvature R is product of reciprocals of maximum curvature radius R1 and minimum curvature radius R2: R = (1 / R1) * (1 / R2). Rotating plane around normal to surface can find both.

surface: triangle

Triangles are three geodesics. Curvature is triangle-angle sum minus pi radians, all divided by triangle area: r = (angle sum - pi) / area.

surface: sphere

Surface curvature is area by surface projection onto a sphere, divided by surface area: r = (projection area) / area.

surface: normals

Curvature is solid angle in radians by normals to surface over surface region, divided by surface area.

surface: volume

Volume can also find curvature.

surface: hexagon

To measure curvature around point, use regular hexagon around the point and measure angles.

bending

If surface bends without stretching, curvature stays constant, because one radius increases and other radius decreases proportionally.

space-time: curvature tensor

In space-time, geodesic deviations, measured along each dimension, are metric second derivative and are second-order differential forms. The result is fourth-order tensor whose matrix coefficients have one covariant and three contravariant indices. For four-dimensional space-time, tensor has 20 independent terms. Metric coefficients are potentials.

Tensor is skew-symmetric and cyclic symmetric and is commutative, with diagonal terms equal zero.

space-time: Weyl tensor

Tensors {Weyl tensor} can measure gravity-field tidal distortion.

space-time: Riemann curvature tensor

At points, total space curvature depends on Ricci tensors and Weyl tensors. Curved space can be Euclidean space {Riemannian manifold} locally. Curved space-time can be Minkowski space {Lorentzian manifold} {pseudo-Riemannian manifold} {semi-Riemannian manifold} locally.

space-time: distance curvature

Second-order tensor {distance curvature} can derive from vector curvature, by contracting matrix coefficients.

space-time: scalar curvature

Tensor trace is scalar invariant. First-order tensors {scalar curvature} can derive from distance curvatures, by contracting to put metric-coefficient second derivatives into linear forms. Scalar curvature is the only such invariant. Physical world has four dimensions, because only such manifold results in curvature invariance.

differential form

Functions have functions {form, mathematics}.

types

Functions are 0-form manifolds.

Exterior derivatives of functions with basis vectors are 1-forms {differential form} {covector} {covariant vector} {1-form, basis}. Tensors operate on 1-forms to give real numbers. Linear operations on vectors can give integral numbers of equally spaced phase surfaces, of de-Broglie particle waves, through which vector passes. Force, energy, momentum, and velocity directional derivatives are 1-form gradients. Anticommutative tensor products and wedge products of two 1-forms are 2-forms. Anticommutative wedge products of three 1-forms are 3-forms. Antisymmetric covariant tensors are k-forms.

algebra

Exterior-derivative wedge products form exterior algebra (Elie Cartan).

dual

p-forms and (n-p)-forms are duals on n-manifolds {Hodge star}. 1-form and vector field are duals and interact to make scalar products. Tangent vector has covector dual.

geodesic tensor

Two manifold points have shortest path {geodesic, tensor}| between them. Geodesic is straightest possible direction between two points.

metric

Quadratic differential linear metric forms can measure geodesic length: ds^2. Geodesic length is sum from points i = 1 to i = m, and from points j = 1 to j = m, of g(i, j) * du(i) * du(j). Coefficients g(i, j) = Du(i) / Du(j), where D are partial derivatives and u are coordinates.

geometry

Geodesic metric defines surface geometry at manifold points.

linear

Using only local operations allows geodesic to be linear.

operator

Geodesic metric operates on vectors to give squared lengths. Squared length can be greater than zero {space-like vector}, less than zero {time-like vector}, or equal to zero {light-like vector}.

space-time

In four-dimensional space-time, particles move along maximum spatial-length lines, which is the shortest possible time as measured in particle reference frame. In flat space-time, geodesics are straight lines. On spheres, geodesics are on great circles.

metric length

For surface, quadratic differential linear form can measure geodesic length {metric, length}|: ds^2.

Ricci tensor

Four-dimensional space has vector curvature with 4x4x4x4 terms, which are linear function-second-derivative combinations. However, terms are in pairs, so four-dimensional vector curvature can contract to 16 independent terms in 4x4 matrices {Ricci tensor}. The other four terms are vector components. Ricci tensor measures volume change, as gravity causes space to contract, and equals energy-momentum tensor. Ricci tensor equals mass-energy density, which is pressure.

MATH>Calculus>Vector>Kinds

hodograph

For vectors with origins at same point, vector ends can represent velocities at different times at the point {hodograph}. Tangent to hodograph is acceleration.

normal

Vectors {normal}| can be perpendicular to curves or surfaces. Normals can point out of convex sides {external normal}. Normals can point out of concave sides {internal normal}.

orthogonal vectors

Vectors {orthogonal vectors}| {orthogonal axes} can be perpendicular. Basis vectors are orthogonal, when axes are independent.

orthonormal

Unit vectors {orthonormal}| can be perpendicular.

position vector

Vectors {position vector} can go from origin to point.

scalar as magnitude

Numbers or variables {scalar}| can have magnitude but no direction.

sensed segment

Line segments {sensed segment} can have beginning end {initial end} and ending end {terminal end}. Sensed segment can be point {point-segment}. Sensed segments or point-segments are vectors and have length and direction.

spinor

Complex-number vectors {spinor}| have rotation around an axis. Specifically, complex-number vectors are second-rank Hermitean spinors. Spinors have direction, amplitude, and frequency. Spinors can be hypercomplex-number vectors. Spinors are like flagpoles, plus flags with lengths, plus orientation-entanglement relations. Though flagpole and flag are like two vectors, spinors are not bivectors, which have real numbers only. Complex-number bivectors are bispinors. Complex-number trivectors are trispinors.

chirality

Spinors have right-handed or left-handed orientation (chirality).

axis

Vectors can rotate around own axis, coordinate axis, or any axis.

quaternions

Quaternions have form a + b*i + c*j + d*k, where a, b, c, and d are real numbers, and i, j, and k are orthogonal unit vectors, so quaternions are vectors in three-dimensional space but with added scalar. Rotating quaternions are real-number spinors. Multiplying quaternions gives i*j = k, j*k = i, k*i = j, j*i = -k, k*j = -i, and i*k = -j, so quaternion multiplication is non-commutative. Multiplying quaternions describes quaternion rotations. Rotation transforms quaternion coordinates {spinor transformation}.

spin matrix

Matrices {spin matrix} describe quaternion and spinor rotations. Spin matrices are scalar products of spinor matrix and rotation matrix: new spin matrix = (rotation matrix) * (old spin matrix) * (rotation-matrix conjugate transpose).

rotation

Spinors reverse sign for 360-degree rotation, because loop cannot shrink to point. Spinors reverse sign twice for 720-degree rotation (4 * pi radians). Rotation sums are vector sums. Two rotations make double-twist that is equivalent to no twist {topological torsion}, because loop can shrink to point.

Complex-plane pi/2 radian rotations rotate pi radians in spherical representations, along oriented great-circle arcs.

If space has n dimensions, rotations are always about axes with n - 2 dimensions. Reflections are always through n - 1 dimension plane. Two reflections through perpendicular planes are equivalent to rotation through pi radians.

history

Elie Cartan discovered spinors [1913] and invented orthogonal-group representation theory.

purposes

Spinors relate geometry, topology, and analysis. Spinors describe fermion and boson spin. Spin matrices (Pauli) and relativistic electron-spin theory (Dirac) use spinors. Spinors are in index theorems for elliptic operators, characteristic number integrability, positive scalar curvature metric existence, twistor spaces, Seiberg-Witten theory, Clifford algebras, spin groups, manifold spin structures, Dirac operators, supersymmetry, four-manifold invariants, and superstring theory.

MATH>Calculus>Analysis

analysis in mathematics

Mathematics branches {analysis, mathematics} {mathematical analysis} can be about theory of real-variable functions and theory of integrals and integral equations.

purposes

Mathematical analysis studies continuous but non-differentiable functions. It studies continuous-function series whose sum is discontinuous. It studies continuous functions that are not piecewise monotonic. It studies functions with bounded derivatives that are not Riemann integrable. It studies curves that are rectifiable, but not by calculus arc-length definition. It studies non-integrable functions that are limits of integrable-function series. It studies Fourier series relations to represented functions.

point density

Number of interval points and number of subinterval points are the same.

Laplace transform

Integral from t = -infinity to t = +infinity of e^(-x * t) * g(t) * dt, where g(t) = (0.5 * i) * (integral from x = a - infinity to x = a + infinity of (e^(x * t))*(f(x)) * dx), where a is large.

integral existence

If interval points are differentiable, function can integrate over interval. Intervals have variable maximum and minimum values. Function f(x) has maximum and minimum over interval. Maximum-M limit minus minimum m times x-change dx goes to zero as dx goes to zero: (M - m) * dx.

series expansion

Functions can be equivalent to series. Function series expansions are in integral-equation theory {convergence of mean} {Lebesgue square integral} {Riesz-Fischer theorem} {moment problem} {Holder inequalities} {strong convergence} {weak convergence} {singular integral equations}.

analytic function

Over intervals, functions can be almost equivalent to power-series functions {Comega-smooth function} {analytic function}. Real analytic functions {Cinfinity-smooth function} can be differentiable at domain points any number of times. Complex analytic functions are complex differentiable, typically only once, at domain points. Complex entire functions are differentiable at all complex-plane finite points. Non-analytic functions are not differentiable at some singularity point or along branch cut.

branch cut

Analytic multiple-valued functions in complex plane can be discontinuous across curve {branch cut} {cut line} {slit, mathematics} {branch line}.

Dirichlet series

Sums, from n = 1 to n = infinity, of a(n) * n^-z, where n is number of terms, a(n) is complex general term, and z is complex number, are number series {Dirichlet series}.

Hessian matrix

Gradient matrices {Hessian matrix} can have first-derivative components. Hessian matrices also have basis-function-parameter second-derivative components, which are typically negligible, because they are like random measurement errors and cancel each other.

MATH>Calculus>Analysis>Orthogonal

orthogonal system

Integral equations relate to complete orthogonal-system theory {orthogonal system}. Functions can expand using orthogonal systems {Fredholm alternative theorem} {complete continuity}.

norm of function

Complex function has complex conjugate {norm, function}. Two infinite complex-number sequences are orthogonal if and only if norm equals zero.

MATH>Calculus>Analysis>Differential Equation

successive substitutions

Using integral equations and initial or boundary values can solve differential equations. Methods {method of successive substitutions} {successive substitutions method} can use iterative substitutions. Methods can use complex-variable functions that are monodrome, monogenic, and holomorphic. Methods can use meromorphic functions.

monodrome

Complex-variable functions can be single-valued {monodrome}.

monogenic

Complex-variable functions can have only one derivative at domain values {monogenic}.

meromorphic function

Single-valued functions {meromorphic function} can be differentiable except at singularities, where they go to infinity. Polynomials can be meromorphic at points {pole, meromorphic function} but cannot have other singularity types. Meromorphic functions are entire-function ratios.

MATH>Calculus>Analysis>Integral

Lebesgue integral

Functions {Lebesgue integral} can have sums of lengths over intervals.

purposes

Lebesgue integrals can integrate discontinuous functions.

finite

Lebesgue integrals can be finite {summable function}. Limit from x = a to x = b of f(x) * cos(n*x) * dx equals zero. Limit from x = a to x = b of f(x) * sin(n*x) * dx equals zero. Therefore, Lebesgue integral can use Fourier series {Riemann-Lebesgue lemma}.

finite: convergence

Functions can have no bound in interval, but Lebesgue integral can converge absolutely.

extensions

Lebesgue-integral extensions include spectral theory {Lebesgue-Stieltjes integral} {ergodic theory} {harmonic analysis} {generalized Fourier analysis}.

Riemann-Darboux integral

Analytic-function sequence limits are integrals {Stieltjes integral, Riemann}. Stieltjes integrals generalize simpler integrals {Riemann-Darboux integral}.

Integrals, from x = a to x = b, of f(x) * dg(x) * dx equal limits of sums, from i = 0 to i = n, of f(e(i)) * (g(x(i + 1)) - g(x(i))), where x(i) are partition intervals and e(i) are inside intervals (x(i), x(i + 1)).

Riemann

Functions {Riemann integrable function} can have no discontinuities or have discontinuities that form measure-zero sets. Riemann integrals are Lebesgue integrable, but Lebesgue integrals can be not Riemann integrable.

Stieltjes integral

Analytic-function sequence limits are integrals {Stieltjes integral}. Stieltjes integrals are generalized Riemann-Darboux integrals. Integrals, from x = a to x = b, of f(x) * dg(x) * dx equals limits of sums, from i = 0 to i = n, of f(e(i)) * (g(x(i+1)) - g(x(i))), where x(i) are partition intervals and e(i) are inside intervals (x(i), x(i+1)).

MATH>Calculus>Analysis>Measure

mathematical measure

Theories {measure theory} can find discontinuous-function magnitudes {mathematical measure} {measure, mathematics}, for quantum mechanics, statistics, and probability.

process

Enclose set points in an open-set interval inside a finite or countably infinite set of non-overlapping intervals {union of non-overlapping denumerable open intervals}. To obtain lower bound {exterior measure}, sum non-overlapping intervals. Use sum to find set-point complement {interior measure}.

measure

If function has bound and is measurable, length, area, or volume is greatest lower bound {greatest exterior measure} and equals least upper bound {least interior measure}.

types

Boolean sigma-algebra can represent discontinuous-function measures. In intervals, Lebesgue generalized ordinary integrals, over discontinuous-function points, can find function values {P-measure}. If Lebesgue integrals are constant, P-measures {Lebesgue measure} are constant.

measurable set

Point-set {measurable set} exterior measure can equal interior measure. If functions are greater than a number, and point sets are measurable, functions are measurable.

MATH>Calculus>Analysis>Method

steepest descent method

Non-linear least-squares parameter estimation methods {steepest descent method} {method of steepest descent} can use points, far from minimum, where first derivative is maximum.

normal equations method

Methods {normal equations method} {method of normal equations} can find function minimum.

inverse-Hessian method

Non-linear least-squares parameter estimation methods {inverse-Hessian method} can use points near minimum, where first derivative equals zero.

Levenberg-Marquardt method

Non-linear least-squares parameter estimation methods {Levenberg-Marquardt method} {Marquardt method} can generalize normal-equations method to find minimum and avoid steepest-descent and inverse-Hessian extremes.

Taking gradient by differentiating eliminates equation constants and so cannot calculate equation-constant magnitude. However, Hessian-matrix components can indicate constant magnitude.

Using scale factor can transform matrix into diagonally dominant matrix. After finding minimum, set scale factor to zero, and compute estimated fitted-parameter standard-error covariance matrix.

MATH>Calculus>Analysis>Operator

continuous additive operator

To generalize solutions involving constants or additions, use axioms A(c*x) = c * A(x) and A(x + y) = A(x) + A(y) {continuous additive operator}.

Einzeltransformation

Linear bounded Hermitean operators can be operators {Einzeltransformation} E such that E-E- = E-, E+E+ = E+, and I = E- + E+. Einzeltransformations are commutative and commute with any operator that commutes with Hermitean operator. Hermitean times E- is greater than zero, and Hermitean times E+ is greater than zero.

fixed-point analysis

In equation A(x) = x, operation or transformation A leaves function or vector constant {fixed-point analysis}. If operator is continuous and operates on n-dimensional spheres, function has at least one fixed point {Brouwer fixed-point theorem}.

projection operator in analysis

Subsets can project {projection operator, set} onto element sets.

MATH>Calculus>Analysis>Operator>Adjoint

adjoint operator in analysis

Operators {adjoint operator} {transposed operator} can find function scalar products, which are linear transformations from one function to another: (A(f1), f2). Adjoint operators can have inverses {self-adjointness}: (A(f1), f2) = (f1, A(f2)). The situation is analogous to the symmetric-integral-equation kernel. (T*f, g) = (f, T*g) and ||T*|| = ||T||, where T* is matrix-T transpose.

Riesz operator

Adjoint-operator theory can apply to operators {Riesz operator} with form I - lambda * V, where lambda is parameter, I is identity operator, and V is complex continuous L^2-space operator.

MATH>Calculus>Analysis>Theorem

content theory

Lengths are closed intervals. The idea of length {theory of content} {content theory} can extend to open intervals. For open intervals, sum of subintervals that enclose points has greatest lower bound {outer content} and sum of polygonal regions makes least upper bound {inner content}. If outer content is less than or equal to inner content, interval has content.

length

If inner content equals outer content, inner content is interval length for one dimension.

additive

For finite number of intervals, sum of disjoint sets with content is sum of set contents {additivity property}.

Hahn-Banach theorem

For adjoint spaces, map of vector space onto line can extend to map of space including line {Hahn-Banach theorem}.

Heine-Borel theorem

In closed intervals (a,b) that have countably infinite interval sets, if a <= x <= b, and x is inside at least one interval, x is inside at least one interval of any finite interval set {Heine-Borel theorem} (Eduard Heine) [1821 to 1881].

projection theorem

Closed space subsets have two unique elements, one in subset and the other orthonormal to every element in subspace {projection theorem} {Riesz representation theorem}.

stationary phase principle

Main contribution to integral is from points at which derivative equals zero {stationary phase principle} {principle of stationary phase}.

Weierstrass-Bolzano theorem

In bounded infinite point sets, a point exists in which any neighborhood has set points {Weierstrass-Bolzano theorem}.

MATH>Calculus>Analysis>Theorem>Spectral Theory

spectral theory

In intervals, analysis can find function-root sets {spectral theory}. Root index is less than its multiplicity. General spectral theory can be for symmetric kernels. Function kernels can have orthonormal eigenfunctions {Hilbert-Schmidt theorem}. Eigenfunction roots {eigenvalue, spectral theory} can be point, band, or continuous spectrum.

spectral radius theorem

Spectral theory can generalize Volterra's method {spectral radius theorem}.

MATH>Calculus>Analysis>Kernel

kernel

Integral from e = a to e = x of K(x,e) * u(e) * de, where K(x,e) are differential equations {kernel, equation} and u(e) equals integral from x = a to x = b of K(x,e) * f(x) * dx, is limiting form of n linear algebraic-equations with n unknowns, as n goes to infinity.

Fredholm equations

Integrals can be from e = a to e = b for K(x,e) * u(e) * de {Fredholm's equations} {Fredholm equations}.

Volterra equations

Integrals can be equal to zero to make homogeneous equations {Volterra's equations} {Volterra equations}.

MATH>Calculus>Analysis>Space

adjoint space

All continuous bounded linear functionals have spaces {dual space} {adjoint space} {Banach space}. Dual-space norms are functional bounds. Duals change L^q to L^p, where q = p / (p - 1). Dual spaces are generalized Hilbert spaces.

complex sequence space

Spaces {complex sequence space} can be equivalent to L^2 spaces of complex-valued, measurable, and square integrable functions. Complex space has inequalities {Schwarz's inequality} {Parseval's inequality}.

function space

Spaces {function space} can have functions as points or distances. Spaces can have finite numbers of elements. Spaces can have infinite numbers of elements, with two limiting elements at interval ends {closed set, function}. Spaces can have no sequence gaps {sequentially compact}. Spaces can have only one limit element {relatively sequentially compact}.

Function spaces can use generalized Pythagorean theorem.

Function spaces can have triangle inequality, Schwarz's inequality, and other inequalities {Bessel's inequality}.

Mutually orthogonal space elements are linearly independent.

Hilbert space in mathematics

Complex abstract spaces {Hilbert space, analysis} can have infinite dimensions.

transformation

Coordinates can transform. Integral from x = a to x = b of K(x,y) * u(x) * dx can transform differentiable function u(x), where K(x,y) are differential equations.

purposes

Differential and integral equation eigenvalue theory is similar to n-dimensional-space linear transformations. Quantum-mechanics equations can use Hilbert-space spectral theory. Observed values are linear symmetric operators in Hilbert space. Linear symmetric energy-operator eigenvalues and eigenfunctions are energy levels. Eigenvalue differences show emitted-light frequencies. Lebesgue integrable functions {square summable function} make spaces similar to Hilbert spaces of sequences. Banach spaces {complete normed vector space} are generalized Hilbert spaces.

linear vector space

Spaces {linear vector space} can have complete orthonormal countable sets. Topology is about equivalencies during continuous motions. Linear vector space can define metric using the norm {strong topology}. Linear vector spaces can have systems of neighborhoods with weak convergence {weak topology}.

Lp space

Banach space includes spaces {L^p space}, of continuous bounded measurable functions, which does not need two orthogonal elements and in which inner product does not define norm.

metric space

Spaces {metric space} can have distances. Integral from x = a to x = b of K(x,y) * u(x) * dx, where K(x,y) are differential equations, can transform distance function u(x) into another distance function.

MATH>Calculus>Analysis>Kinds

combinatorial analysis

Permutations, combinations, binomial theorem, magic squares, and partition theory can combine into one subject {combinatorial analysis}.

functional analysis

Continuous functions, convergences, and limits can combine into one subject {functional analysis}. Analysis includes infinite series, ordinary differential equations, partial differential equations, differential geometry, calculus, and calculus of variations. Analysis excludes plane geometry, solid geometry, and computational methods. Analysis uses arithmetic, variable, function, continuity, differentiability, integrability, limits, infinitesimals, infinite, least upper bound, uniformity, convergence, and fundamental theorem of calculus.

purposes

Functional analysis can be for generalized moment problem, statistical mechanics, fixed-point theorems, partial-differential-equation existence and uniqueness theorems, calculus of variations, and continuous compact-group representation. Linear functional analysis can study integral equations.

functional

Functions of functions {functional} map function to number. Functionals can find areas of products of two functions, over intervals. Functionals can evaluate functions at points. Functionals have generalized derivatives that are also functionals, but can have singularities.

linear functional analysis

Sum from p = 1 to p = infinity of (z(p) * Z(p))^0.5, where Z(p) is z(p) complex conjugate, can study calculus of variations {linear functional analysis}.

standard analysis

Analysis {standard analysis} can use limits and exhaustion method. Standard analysis and nonstandard analysis use same language and rules, but interpretation is different.

nonstandard analysis

Analysis {nonstandard analysis} can use infinitesimals that can never get large. In nonstandard analysis, numbers, metrics, and spaces always have nearby values.

contradiction

Non-standard analysis can introduce contradictions, because added infinitesimals do not necessarily stay small. Infinitesimals can violate Archimedes principle {non-Archimedian}.

theorem

For proposition sets, if finite proposition subsets are true in standard analysis, whole proposition set is true in nonstandard analysis {compactness theorem}.