Outline of Algebra
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MATH>Algebra

algebra in math

Mathematics {algebra} can be about functions and equations.

MATH>Algebra>Term

term

A coefficient can multiply one or more variables raised to powers {term, algebra}. Terms can add.

simplification

To simplify terms with parentheses, first remove exponents outside parentheses by multiplying exponent by all term exponents, including exponents of 1. After removing parentheses, multiply term factors. Make term have fewest variables and coefficients.

If term has more than one constant or repeats same variable, first multiply coefficients, including signs, to obtain one coefficient. Then, for all variables, add exponents.

addition

To add or subtract terms, first put terms in simplest form. Then add coefficients of terms that have same variables with same exponents.

cosa

unknowns {cosa}.

similar term

Terms {similar term} {like term} can differ only in numerical or literal coefficient.

surd

Expressions can have irrational numbers {surd}. Expressions {pure surd} can have irrational numbers in all terms. Expressions {entire surd} can have no rational factors.

MATH>Algebra>Term>Coefficient

coefficient of term

Numbers {coefficient} {constant, algebra}, or letters that equal constants, can multiply variables. Coefficients have no exponent.

literal coefficient

Coefficients {literal coefficient} can use letters, instead of numbers.

MATH>Algebra>Term>Variable

variable in algebra

Letters {variable, algebra} can have values. Variables can have constant or variable exponents.

power of variable

Variables {base, exponent} can have exponents {exponent, variable} {power, variable}. To multiply variable with exponent with same variable with exponent, add exponents: x^3 * x^2 = x^5. To divide variable with exponent by same variable with exponent, subtract exponents: x^3 / x^2 = x^1. To raise variable with exponent to power, multiply exponent and power: (e^x)^3 = e^(3*x).

MATH>Algebra>Term>Sum

monomial

Expressions {monomial} can have one term.

binomial

Sums can have two terms {binomial}.

trinomial

Sums can have three terms {trinomial}.

polynomial

Sums can have more than one term {polynomial}.

MATH>Algebra>Kinds

Banach algebra

Complex-number algebras {Banach algebra} can have norms. Norms can be absolute values. Product norm is less than or equal to product of norms. Real numbers, quaternions, or complex numbers form Banach algebra for multiplication when norm is absolute value. Banach algebra is associative. It can be commutative or non-commutative. It can have or not have identity element.

division algebra

Only real numbers, complex numbers, quaternions, and octonions allow division {division algebra}.

MATH>Algebra>Equation

equation general

Two mathematical expressions can be equal {equation, algebra}. Mathematical expression can be less than or greater than another mathematical expression {inequality, mathematics}.

fundamental theorem of algebra

Polynomial equations have at least one solution {fundamental theorem of algebra}. The fundamental theorem of algebra requires complex numbers.

bilinear equation

Linear equations {bilinear equation} can have products of two variables: c*x*y + b = 0, where x and y are variables to first power, and c and b are constants. Bilinear equations can be sums of indexed variables: a11 * x1 * y1 + a12 * x1 * y2 + ... + aNN * aN * yN, where aNN are constants, and xN and yN are indexed variables.

MATH>Algebra>Equation>Graph

asymptote of graph

Graphs can approach limiting slope {asymptote, graph}|, as independent variable approaches infinity or negative infinity.

unit circle

Circles {unit circle} can have center at origin of Cartesian coordinates and radius one unit.

angle

Unit-circle points define angle between radius and positive x-axis.

Counterclockwise angle is positive. 90 degrees equals pi/2 radians. 180 degrees equals pi radians. 270 degrees equals 3 * pi/2 radians.

Clockwise angle is negative. -90 degrees equals -pi/2 radians. -180 degrees equals -pi radians. -270 degrees equals -3 * pi/2 radians.

coordinates

Unit-circle point (x,y) relates to angle A by trigonometric functions: x = cos(A) and y = sin(A). Coordinates x and y relate: x^2 + y^2 = 1, because radius = 1.

MATH>Algebra>Equation>Solution

root of equation

Equation solutions {root, equation}| {zero, equation} are variable values that make equation true {solution, equation} {equation, solving}. Roots {null, equation} can be non-existent or zero. Equation-solution number equals equation degree. Two or more solutions can be equal {multiplicity, solution}.

order

Equations have highest exponent. Roots can be highest radical {order, radical} {radical, order}.

homogeneity

Equations can equal zero: $f(x) = 0$. Homogeneous equations have solutions.

polynomial

If polynomials have real-number coefficients, one factor is coefficient of highest-power term times product of polynomials of form $(x - r)$ or $(x^2 - (r + r')*x + r*(r'))$, where r is root and r' is root complex conjugate. Root complex conjugate is also a root.

complex number

If x and n are real, equation $x^n = -1$ solutions are complex numbers {roots of unity} {nth roots of unity}. For $x^2 = -1$, $x = i$. For $x^3 = -1$, $x = i^{(2/3)}$. If w and z are complex, equation $w^z = 1$ solutions are $\log(1) = 0, 2 * pi * i, 4 * pi * i, 6 * pi * i, 8 * pi * i, ...$

In unit circle, solutions are regular-polygon vertices, forming complex-number cyclic group Zn, a finite multiplicative group.

determinate equation

If equations {determinate equation} have only one unknown variable, equation has a numerical solution. If equation {indeterminate equation} has more than one unknown variable, equation has solution in terms of unknowns. If unknowns number equals equation number, equation system has numerical solutions. If unknowns number is more than equation number, equation system has solutions in terms of unknowns.

evaluation of expression

Expressions can have numeric values {evaluation, expression}.

checking solution

To check solution steps {checking solution}, do additions and multiplications forward and backward, making sure that sign is correct. For $(x - 3) / (x + 2) = x / (x - 2)$, $x^2 - 2*x - 3*x + 6 = x^2 + 2$, so $7x = 6$.

discriminant

For general quadratic-equation solution, quantity {discriminant, solution} under square root sign is $b^2 - 4*a*c$.

extraneous solution

If unknowns number is less than equation number, solutions {extraneous solution}| can depend on other solutions.

repeated root

For polynomials divisible by $(x - a)^n$, root a {repeated root} repeats n times.

signs and roots rule

If polynomial equals zero and has two positive or negative terms in succession, at least one root is negative {signs and roots rule} {rule of signs and roots}. If polynomial equals zero and has positive term succeeding negative term, or negative term succeeding positive term, at least one root is positive.

MATH>Algebra>Equation>Solution>Methods

extrapolation

If you know two function points, two points make a line, and you can use the linear function to estimate function values {extrapolation}| for independent-variable values greater than the larger, or less than the smaller, of the point independent-variable values.

interpolation

If you know two function points, two points make a line, and you can use the linear function to estimate function values {interpolation}| for independent-variable values between the point independent-variable values.

isolate variable

All terms with unknown variable can be on left or right equation side {isolate variable}.

process

Remove exponents outside parentheses by multiplying.

Remove all fractions and divisions by multiplying out all denominators. Factor denominator and cancel factors, divide into numerator to get quotient, or multiply both equation sides by denominator.

Remove parentheses by performing all multiplications, to make sum of terms.

Make irrational numbers or variables rational by taking both equation sides to power.

Multiply repeated variables in terms to make one variable.

Add similar terms.

Add all constants.

Put all terms containing unknown variable on one equation side.

roots

Assume left-side expression has unknown variable. Find left-side-expression roots by polynomial factoring or other method. Find right-side-expression roots. For whole equation, root is left-side-expression root minus right-side-expression root.

linearize

If h is small compared to x, change (x + h)^n to x + n*h to remove exponent and make linear {linearize}.

MATH>Algebra>Equation>System

system of linear equations

Equation sets {linear-equation system} {system of linear equations} can have only variables to first power. To find roots, use determinant laws. To determine points and slopes, use determinant laws. To rearrange equations, use matrix laws.

determinative system

In equation systems {determinative system} {consistent system} {simultaneous system}, number of independent equations can equal number of variables, and all variables have numerical solutions. Number of independent equations can be less than number of variables {inconsistent system}, so not all variables have numerical solutions. If some equations are equivalent to others {dependent system}, not all variables have numerical solutions. Number of independent equations can be more than number of variables {overdetermined system}.

triangular form

For linear-equation systems {triangular form}, first linear equation can have only first variable, second equation can have only first and second variables, and so on.

MATH>Algebra>Equation>System>Solution

Cramer rule

For linear-equation system, variable equals determinant value divided by resultant-determinant value {Cramer's rule} {Cramer rule}.

dialytic method

If two equations contain unknown raised to power, eliminate unknown from both equations by substitution {dialytic method}.

elimination from equation

To eliminate a term {elimination, equation}, subtract one equation from another equation. If needed, multiply equation by coefficient or variable before subtracting.

Gauss-Jordan elimination

Dividing equations by coefficients and subtracting equations {Gauss-Jordan elimination} can solve equation systems.

process

Divide first row by first-variable coefficient {pivot element}, so first-variable coefficient is one. For other rows, subtract multiple of first row to make first-variable coefficient equal zero, and replace row with resulting row.

Divide new second row by second-variable coefficient, so second-variable coefficient is one. For other rows, subtract multiple of second row to make second-variable coefficient equal zero, and replace row with resulting row.

Follow same steps for all rows. Use pivoting to avoid dividing by zero.

result

All rows begin with variable with coefficient equal one. All rows begin with different variables: row n begins with nth variable.

multiplier method

To solve equation systems, multiply {multiplier method} one equation by a scalar to make unknown's coefficient the same as unknown's coefficient in a second equation. Then subtract first equation from second equation to eliminate term with the unknown. Multiplier method does not change resultant determinant.

pivoting in equation solving

Interchanging rows {partial pivoting} or interchanging rows and columns {full pivoting} can put term to eliminate on the diagonal {pivoting in equation solving}. Typically, pivot is largest term.

power function of linear equations

To solve linear-equation systems, sum all linear-equation powers to derive a power function and then find power-function minimum {power function, linear equations}.

substitution in equation

To solve equation systems, rearrange equation terms to have only one variable on equation left side {substitution, equation}. In second equation containing that variable, substitute first-equation right side for variable, to eliminate variable from second equation. This is an example of replacing whole by sum of its parts.

MATH>Algebra>Equation>System>Matrix

matrix

Numbers, terms, and vectors can be in arrays {matrix, mathematics}. Two-dimensional matrices have vertical positions {column, matrix}, horizontal positions {row, matrix}, and elements {cell, matrix}. Infinite matrices can have any number of dimensions, with any number of elements, as in quantum mechanics.

notation

Matrix notation is braces.

examples

One-element matrix is scalar. One-row matrix is vector.

multiplication

Matrices have products of scalars, vectors, and matrices.

purposes

Matrix elements can represent relations between set members. Matrices can be truth-tables, with element T or F listed for statement pairs. Propositions can be matrices in Boolean algebra form.

Matrices can be ordered-set components. Sequences can be n-dimensional matrices.

Matrices can represent states and operations of mathematical groups, state spaces, and symmetries. Matrices can represent particle-pair spin states.

Matrices can represent graphs. Rows and columns represent nodes. Elements are connection values between nodes.

Matrices model linear equations. Quadratic expressions use matrices to find moments of inertia. Product of solution-matrix transpose and coefficient matrix and solution matrix can find linear-equation solutions.

element of matrix

Matrices have cell values {element, matrix}.

order of matrix

Matrices have number of dimensions {order, matrix}. Scalars have order zero. Vectors have order one. Two-dimensional matrices have order two.

rank of matrix

Matrices have maximum row or column number {rank, matrix}.

MATH>Algebra>Equation>System>Matrix>Characteristic

characteristic equation

For linear equations, matrix equations {characteristic equation, matrix} can set matrix determinant minus x times unit-matrix determinant equal to zero-matrix determinant: |M| - x * |1| = |0|. Solving for x gives equation roots.

trace of matrix

Characteristic-equation-root sums {trace, matrix} can be matrix parameters. Unitary matrices have invariant traces {character function} {group character} {character, matrix} that characterize the mathematical group that the matrix represents.

characteristic value

Coefficient-matrix A and solution-matrix X product can have a factor {lambda} {characteristic value}: A*X = lambda * X.

MATH>Algebra>Equation>System>Matrix>Operations

matrix addition

Adding corresponding elements adds matrices {matrix addition}. Adding vectors is an example. Summing matrices is like adding one effect to another effect to get total effect.

matrix multiplication

To multiply matrices {matrix multiplication} {matrix dot product}, multiply each row by each column. Matrix with m columns and n rows times matrix with n columns and p rows makes matrix of m columns and p rows. First-matrix rows and second-matrix columns must have same rank. For 1x1 matrices [a11] and [b11], matrix dot product is [a11*b11]. For 2x2 matrices [a11 a12 / a21 a22] and [b11 b12 / b21 b22], matrix dot product is [a11*b11 + a11*b21 a12*b12 + a12*b22 / a21*b11 + a21*b21 a22*b12 + a22*b22]. For example, [1 2 / 3 4] . [5 4 / 3 2] = [1*5+1*3 2*4+2*2 / 3*5+3*3 4*4+4*2].

vector

Vector dot products are matrix multiplications of one-row 1xN matrix with one-column Nx1 matrix.

properties

Matrix multiplication is not commutative but is associative.

purposes

Multiplying matrices indicates results of interactions between two effects. Squaring matrix is like repeating operation.

matrix cross product

Cross products {matrix cross product} of two square matrices indicate interactions between set-A and set-B members: A x B. Matrix cross products can find extensive quantities, such as area, from intensive quantities, such as vector distances. Matrix cross products are differences between matrix dot product and reverse matrix dot product: A x B = (A . B) - (B . A). Only square matrices can have matrix cross products. Matrix cross products find square matrices. For 1x1 matrices [a11] and [b11], matrix cross product is [a11*b11 - b11*a11] = [0]. For 2x2 matrices [a11 a12 / a21 a22] and [b11 b12 / b21 b22], matrix cross product is [a11*b11 + a11*b21 a12*b12 + a12*b22 / a21*b11 + a21*b21 a22*b12 + a22*b22] - [b11*a11 + b11*a21 b12*a12 + b12*a22 / b21*a11 + b21*a21 b22*a12 + b22*a22] = [a11*b11 + a11*b21 - b11*a11 - b11*a21 a12*b12 + a12*b22 - b12*a12 - b12*a22 / a21*b11 + a21*b21 - b21*a11 - b21*a21 a22*b12 + a22*b22 - b22*a12 - b22*a22] = [a11*b21 - b11*a21 a12*b22 - b12*a22 / a21*b11 - b21*a11 a22*b12 - b22*a12]. If both matrices are the same, matrix cross product is zero matrix: A x A = 0. Matrix cross products are not commutative: A x B = (A . B) - (B . A) <> (B . A) - (A . B) = B x A.

Cayley-Hamilton theorem

If M is a square matrix and another matrix is equivalent to M, their difference is zero matrix {Cayley-Hamilton theorem}. Theorem helps find characteristic equation.

conjugate transpose

For non-unitary matrices, replacing each matrix element by its complex conjugate and transposing the matrix {Hermitean operation, transposing} makes the same matrix {conjugate transpose}. For unitary matrices, matrix conjugate transpose is matrix inverse.

equivalence operation

Interchanging any two matrix rows does not change matrix meaning {equivalence operation}. Multiplying all elements in row by non-zero number does not change matrix meaning. Replacing row by sum of itself and another row does not change matrix meaning.

Hermitean operation

Replacing each matrix element by its complex conjugate and transposing matrix {Hermitean operation, matrix} makes a matrix. Hermitean operators follow general eigenvalue theory, where (R(f),g) = (f,R(g)) and R is linear.

trace as sum

Square-matrix diagonal elements have a sum {trace, sum}. Matrix-product trace is first-matrix trace times second-matrix trace. Finding matrix-product traces is commutative: trace(A . B) = trace(B . A).

MATH>Algebra>Equation>System>Matrix>Kinds

cofactor matrix

Matrices {cofactor matrix} can represent linear-equation systems. Columns are variables plus one column for constant. Rows are equations. Elements are variable coefficients. Alternatively, elements can be variables, and columns can be variable coefficients.

augmented matrix

Variable-coefficient and constant matrices {augmented matrix} can represent linear-equation systems.

adjoint matrix

Matrices have associated matrix {adjoint matrix} that replaces each element by its cofactor.

complex-number matrix

Complex numbers are equivalent to 2x2 real-number matrices {complex-number matrix} whose diagonal elements are equal and whose off-diagonal elements are equal but opposite in sign: a + b*i = (a b / -b a), where / indicates row end. For example, i equals (0 1 / -1 0), and 1 equals (1 0 / 0 1). Complex numbers and 2x2 real-number matrices have the same results under addition and multiplication, and the determinant of 2x2 real-number matrices equals the absolute value of their complex numbers.

inverse matrix

Non-singular matrices have associated matrices {inverse matrix} that are reciprocals of determinant times adjoint matrix. To find matrix inverse, replace each element by its cofactor divided by matrix determinant. Matrix inverse can solve equations. Linear-equation system has coefficient matrix A, solution matrix X, and cofactor matrix B. A-inverse times A times X equals B: A^-1 * A * X = B. Solution matrix X equals coefficient-matrix A inverse times adjoint matrix B: X = A^-1 * B.

Jordan canonical form

Matrices can have standard forms {Jordan canonical form}.

normal form

Matrices can be the identity matrix {normal form, matrix}.

quadratic form

Linear-equation systems have variable-coefficient matrices {quadratic form} and solution matrices. Solution matrix X transpose times variable-coefficient matrix A times solution matrix X equals bivariate sum with three coefficients: (transpose of X) * A * X = a*x*x + b*x*y + c*y*y.

singular matrix

Matrices {singular matrix} can have determinant equal zero. Matrices {non-singular matrix} can have determinant not equal zero.

square matrix

Matrices {square matrix} can have same number of rows and columns.

transpose matrix

Square matrices {transpose matrix} {transverse matrix} can interchange rows and columns. Matrix transpose can be same as matrix {symmetric matrix}, negative of matrix {skew symmetric matrix}, or conjugate of matrix {conjugate matrix}. Transposition can define square matrices {Hermitean matrix} {skew Hermitean matrix}. Adjoint matrices have transposes. Inverse transverse-matrix can equal matrix {orthogonal matrix}.

triangular matrix

If principal diagonal is not all zeroes, matrices {triangular matrix} can transform to have only zeroes on left or right of principal diagonal.

unimodular matrix

Square matrices {unimodular matrix} can have determinant equal one.

unitary matrix

Matrices {unitary matrix} can have ones on diagonal and zeroes everywhere else. Products of two unitary matrices make a unitary matrix.

group

Mathematical groups have representations {representational theory, group} as sets of same-order square unitary matrices, whose determinants equal one. For groups, square unitary matrices make an orthonormal basis-vector set.

group: trace

Unitary-matrix traces are invariant under transformation. Traces characterize the mathematical group. All class members have same trace. Different class characters are orthogonal.

MATH>Algebra>Equation>System>Determinant

determinant in mathematics

Matrices define square element arrays {determinant, equation}|. Determinant symbol uses vertical lines at array sides: |A|. For square matrix, determinant elements are same as matrix elements. Second-order square matrices have rows "a b" and "c d": {a b / c d}, where / denotes row end. Determinant is |a b / c d|. Second-order square matrices have four elements. Third-order square matrices have nine elements. Fourth-order square matrices have 16 elements.

value

Determinants have scalar values, which are like area. To find determinant value, multiply each element of first column or first row by its signed minor. Add all products.

value: dependence

If a determinant row is a linear combination of other rows, determinant value equals zero.

value: triangular matrix

For triangular matrices, determinant value is product of principal-diagonal elements.

inverse

If matrix has determinant value zero, matrix is singular and has no inverse.

equation system

Equation systems have coefficient and constant arrays. Resultant determinant has variable coefficients: 2*x + 3*y = 0 and 4*x + 5*y = 0 goes to |2 3 / 4 5| = 2*5 + -3*4 = 2*5 + -4*3 = -2. Variables have determinants. Constants column replaces variable-coefficient column. For variable x, |0 3 / 0 5| = 0*5 + -3*0 = 0*5 + -0*3 = 0.

Determinative non-homogeneous linear-equation systems have determinant value not equal zero. Determinative homogeneous systems of linear equations have determinant value zero. To find variable values, use coefficient and constant determinant.

minor

Determinant elements can have subdeterminants {minor, determinant} containing elements that are not in same row and column. For determinant with rows "a b" and "c d", element-a minor is d, because a is in first row and column, and d is not in first row and not in first column. The smallest minor is one element.

cofactor of determinant

Minors {signed minor} {cofactor}| can have sign. Sign depends on sum of element row and column positions. If element is in row and column whose sum is odd, sign is -1. If element is in row and column whose sum is even, sign is +1. For determinant |a b / c d|, a's signed minor is +d, because element a has row 1 and column 1, which sum to 2, which is even. b's signed minor is -c, because element b has row 1 and column 2, which sum to 3, which is odd. Therefore, determinant value is a*d - b*c.

modulus of determinant

Determinants can have squares, cubes, and higher powers {modulus, determinant}.

rank of determinant

Determinants have a number of rows {rank, determinant}. Determinants have same number of columns, because they are square. Matrix rank is biggest non-zero-determinant number of rows.

Sarrus rule

To find determinant value, copy determinant {Sarrus rule} {rule of Sarrus}. Write all determinant rows, except last row, below copy. Multiply elements on each diagonal. Change sign on ascending diagonals, going down to right, but not sign on descending diagonals, going up to right. Add terms.

MATH>Algebra>Equation>System>Determinant>Types

Hessian determinant

For a matrix representing linear homogeneous equations, partial-derivative coefficient determinant {Hessian determinant} indicates inflection points.

Jacobian determinant

Determinants {Jacobian determinant} {functional determinant} can be for coordinate transformations: double integral of F(x,y) * dx * dy = double integral of G(u,v) * determinant(fu, fv, gu, gv) * du * dv. Jacobian determinants have two rows of partial derivatives, one row for F(x,y) and one row for G(u,v). Jacobian determines scalar for unit vectors. Jacobians can determine normal vectors at function intersections.

similar determinant

Product of determinant inverse and second determinant B and first determinant A calculates third determinant C {similar determinant}, which is similar to second determinant: A^-1 * B * A = C.

MATH>Algebra>Equation>Kinds

cyclotomic equation

Equation x^p - 1 = 0 {cyclotomic equation}, where p is prime number and x is independent variable, constructs polygons in circles. Polygons in circles have rotation-symmetry groups.

defective equation

Lower-order equations {defective equation} can derive from equations.

Diophantine equation

Polynomial equations {Diophantine equation} can have integer coefficients. It can have integer solutions. If system of Diophantine equations has nine variables, no algorithm can decide if system has integer solutions.

homogeneous equation

Linear equations {homogeneous equation}| can have constant equal zero.

parametric equation

Equations {parametric equation} can depend on one variable. For example, line has parametric equations x = x1 + t*A and y = y1 + t*B, where t is parameter, A and B are constants, x and y are coordinates, and (x1,y1) is point on line. Line slope is B/A, if A is not 0. If A = 0, line is parallel to y-axis.

periodic equation

Wave equation {periodic equation} repeats at regular intervals: y = A * sin(a*x + b) or y = A * cos(a*x + b) or y = A * sin(a*x + b) + B * cos(c*x + d), where A is maximum y amplitude, a and b are constants, and x is independent variable. Phase angle is a*x + b. Angle phase shift from 0 is -b/a.

proportion

Two ratios can be equal {proportion} {analogy, mathematics}: a/b = c/d. In proportion, variable can be in numerator and another variable can be in other numerator {direct variation} {direct proportion}: x/3 = y/4. In proportions, product of two variables can be in numerator {inverse variation} {inverse proportion}: x*y = 4. Direct or inverse proportion can have a constant {proportionality constant}: c * (x/3) = y/4 or c * (x*y) = 4.

Proportions can change to product equalities by multiplying {cross-multiply} both equation sides by denominator product: x/3 = y/4 goes to 12 * x/3 = 12 * y/4 = 4*x = 3*y.

symmetric equation

Two-variable equations {symmetric equation} can interchange x and y to make same equation.

MATH>Algebra>Equation>Kinds>Degree

quadratic equation

Equations {quadratic equation} {second-degree equation} {quadric equation} can have one variable raised to second power. Quadratic equations have form a*x^2 + b*x + c = 0. Solution is x = (-b + (b^2 - 4*a*c)^0.5)/(2*a) and x = (-b - (b^2 - 4*a*c)^0.5)/(2*a). Quantity under square root is b^2 - 4*a*c {discriminant, quadratic equation}.

square

If quadratic equation has form a^2 * x^2 + b * x = c, add (b/(2*a))^2 to both equation sides {completing the square}. Factor left side: (a*x + b/(2*a))^2 = c. Take square root of both sides: a*x + b/(2*a) = c^0.5. Solve for x: x = (-b/(2*a) + c^0.5)/a.

factoring

If quadratic equation has form a*(x - r1)*(x - r2) = 0, then x = r1, x = r2, a = r1 + r2, and r1 * r2 = c/a.

graph

Quadratic-equation graph has U-shape.

cubic equation

Equations {cubic equation} can have variable raised to third power. General cubic equation has a solution.

quartic equation

Equations {quartic equation} can have variable raised to fourth power. General quartic equation has a solution.

n-tic equation

General degree-higher-than-four equations {n-tic equation} do not have solutions.

MATH>Algebra>Equation>Kinds>Line

linear equation

Equations {linear equation} {first-degree equation} can have variables raised only to first power.

graph

Linear-equation graph is a straight line.

form

Standard form is a1 * x1 + a2 * x2 + ... + aN * xN = b, where b is constant, a1 to aN are coefficients, and x1 to xN are variables.

homogeneous

Linear homogeneous equations equal zero.

one variable

Linear equation can have only one variable: a1 * x1 = b. Straight line can have point where line intercepts y-axis {y-intercept} and point where line intercepts x-axis {x-intercept}. Straight line inclines to x-axis {slope, line}.

general form of linear equation

Linear equation can have form a*y + b*x = c {general form}. x = (c - a*y) / b. Slope is -b/a. y-intercept is c/a.

intercept form of linear equation

Linear equation can have form x/a + y/b = 1 {intercept form, line}. y-intercept is a. x-intercept is b.

normal equation of linear equation

Straight lines in planes can have length p. x * cos(A) + y * sin(A) - p = 0 {normal equation}, where A is angle that perpendicular from origin (0,0) to straight line makes with positive x-axis.

point-slope form of linear equation

Linear equation can have form y - A = m * (x - B) {point-slope form}. (B,A) is line point. Slope is m.

slope-intercept form of linear equation

Linear equation can have form y = m*x + b {slope-intercept form}. x = (y - b) / m. Slope is m. y-intercept is b.

MATH>Algebra>Formal System

algebra as system

Sets, such as numbers, can have two operations, such as addition and multiplication {algebra, system} {formal system, algebra}. Operations map elements, or element pairs, to elements: 1 + 2 = 3. Arithmetic is algebra. Elements are real numbers, and operations are addition and multiplication.

associative law of addition

In arithmetic, a + (b + c) = (a + b) + c {associative law of addition}.

associative law of multiplication

In arithmetic, a * (b * c) = (a * b) * c {associative law of multiplication}.

commutative law of addition

In arithmetic, a + b = b + a {commutative law of addition}.

commutative law of multiplication

In arithmetic, a * b = b * a {commutative law of multiplication}.

distributive law

In arithmetic, a * (b + c) = a*b + a*c {distributive law}.

identity element of system

A number {identity element, addition} can sum with another number to make second number. A number {identity element, multiplication} can multiply with another number to make second number. In arithmetic, 0 is additive identity and 1 is multiplicative identity.

inverse element of system

In arithmetic, numbers has numbers {additive inverse} {inverse element, algebra} that can add to make additive identity element: n + -n = 0. Numbers have numbers {multiplicative inverse} that can multiply to make multiplicative-identity element: n * (1/n) = 1.

Jacobi identity

In non-associative algebras, association-axiom replacement {Jacobi identity} can be (a, (b, c)) + (b, (c, a)) + (c, (a, b)) = 0. Commutative-axiom replacement can be (a, b) = -(b, a).

MATH>Algebra>Function

mathematical function

In algebraic-operation sequences {mathematical function} {function, mathematics}, every domain value can result in only one range value. Defined variable {dependent variable, function} equals function of original variable {independent variable, function}: y = f(x). Functions can have more than one independent variable: y = f(x1, x2, ...).

domain

Independent variable value is in a possible-value set.

range

Dependent variable value is in a possible-value set.

mapping

Domain elements correspond to one range element {image, range} {mapping, range}: x -> y.

one-to-one correspondence

Domain element can correspond to only one range element, and range elements can correspond to only one domain element.

relation

In algebraic-operation sequences {relation, function}, every domain value can result in one or more range values. Independent-variable values can result in more than one dependent-variable value. Relations are about order, symmetry, transitivity, reflexivity, equivalence, material implication, inclusion, one-to-one correspondence, and many-to-one correspondence relations.

relation: explicit

Two variables can directly relate {explicit relation} {explicit function}. Example is y = 3*x.

relation: implicit

Two variables can relate to same parameter {implicit relation} {implicit function, algebra}. Example is x = t and y = 3*t, so y = 3*x.

relation: reflexive

Symbol, concepts, or sentences can refer to itself. Example is "This sentence is true." Relations can relate domain elements to themselves {reflexive relation}. Example is the equality relation x = x.

relation: anti-reflexive

Relations can relate no domain element to itself {anti-reflexive relation}. Example is the inequality relation: y > x.

relation: non-reflexive

One or more domain elements can not relate to itself {non-reflexive relation}.

relation: recursive

Relations can repeatedly apply algebraic-operation sequences to domain elements to make successive terms {recursive relation} {recurring sequence} {recurring series}: for example, x, x + x = 2*x, 2*x + x = 3*x, ..., (n - 1)*x + x = n*x. In recursive relations, nth term has coefficient {scale, relation}.

relation: transitive

Relations {transitive relation} can preserve value order. Example is if a > b and b > c, then a > c.

variation: monotonic decreasing

Quantities can always decrease or stay the same {monotonic decreasing}.

variation: monotonic increasing

Quantities can always increase or stay the same {monotonic increasing}.

variation: joint

Variables can vary directly with product of other variables {joint variation}.

variation: combined

Variables can relate to expressions of other variables {combined variation}.

variation: invariance

After transformation, functions can result in same product as before. By energy conservation, functions that calculate energy have invariance. Invariants are covariants of order zero.

generalized function

Discontinuous functions {generalized function} can have derivatives.

linear function operations

Sum of a * f(x) + b * g(x) equals a * (sum of f(x)) + b * (sum of g(x)) {linear function, operations}. Sum from x = m to x = n of f(x) equals (sum from x = m to x = p of f(x)) + (sum from x = p + 1 to x = n of f(x)), where m < p < n. Sum from x = m to x = m of f(x) equals f(m). Sum from k = 1 to k = n of k equals (n^2 + n) / 2. Sum from k = 1 to k = n of k^2 equals (2 * n^3 + 3 * n^2 + n) / 6.

moment

Difference between value and mean or another value can have power {moment, function}: (x - u)^2. First moment is first power of difference: x - u. Second moment {variance, moment} is second power of difference: (x - u)^2. Third moment {skewness, moment} is third power of difference: (x - u)^3. Fourth moment {kurtosis, moment} is fourth power of difference: (x - u)^4.

singularity in function

Function can have points {singularity, function} where it has multiple values {degeneracy, function} and so is not a function.

boundary

Line can have all or many singular points {natural boundary}.

types

Polynomial functions can go to infinity {pole, singularity}. Function main term can have form ln(x) {logarithmic singularity} {logarithmic branch point}. Function can go to infinity but have bound {removable singularity}. Pole, logarithmic singularity, removable singularity, and essential singularity have no other singularities nearby {isolated singularity}. f(x) * (x - c)^n, where n is integer greater than 0, can be not differentiable {essential singularity}.

MATH>Algebra>Function>Range

domain of function

Independent variables have possible values {domain, variable}|.

range of function

Dependent variables have possible values {range, variable}|.

critical value

Function values {critical value} at endpoint, undefined point, and optimum are the most-important values.

intermediate value theorem

For function over interval, domain value gives range value that is between range values at interval endpoints {intermediate value theorem}.

correspondence rule

Domain members can map to only one range member {correspondence rule, function} {rule of correspondence}, because function has only one value.

equipotent set

Function and inverse can have same number of elements in domain and range {equipotent set}.

MATH>Algebra>Function>Map

mapping of function

If domain elements correspond to only one range element, and range elements correspond to only one domain element, domain and range elements have one-to-one correspondence {mapping, function}. Functions and their inverse functions map.

conformal map

Two functions can map {conformal map} in an ordered sequence, with no overlaps or gaps. Angles are the same as in Euclidean space. Shapes stay the same, but sizes change.

Riemann mapping theorem

Complex-plane closed region, bounded by closed loop that has no intersections, can holomorphically map onto complex-plane unit disc {Riemann mapping theorem}. Two simply connected planes can conformally map one-to-one.

One inner and one boundary point of one plane are in other plane. Transformations on unit circles through z = -1 makes airfoil shapes {Zhoukowski airfoil transformation} {Joukowski airfoil transformation}.

MATH>Algebra>Function>Transformation

analytic continuation

Starting with holomorphic function in complex-plane region, the function can extend to other domains by moving along path to points that allow overlapping regions {analytic continuation}. Different paths result in different extensions.

contravariance

Tensors can project onto coordinate systems, with basis vectors, to find coefficients {contravariance}|. Spatial-coordinate differentials dx^i make simplest contravariant vector {rank one tensor}. Contravariant coefficients contracted with metric tensor give covariant coefficients.

covariance

Transforming functions can result in same product {covariance}| times a constant or a power of determinant {modulus, covariance}. For tensors, covariance transforms basis-vector coefficients into other basis-vector coefficients.

family of functions

Function groups {family of functions} can differ by algebraic parameter.

invariance of function

Transforming functions can have same results {invariance, function}. Energy conservation requires that total energy be invariant. Functions can remain the same under linear transformations {algebraic invariant}. Invariants are covariants of order zero.

MATH>Algebra>Function>Kinds

entire function

In plane regions, functions {entire function} can be single-valued and have no singularities.

greatest integer function

Functions {greatest integer function} can have range equal nearest lower integer when domain is a real number.

identity function

Functions {identity function} can have range equal to domain.

infinite function

As independent variable approaches value, function {infinite function} can approach +infinity, -infinity, or (infinity)^-1 = 0.

inverse function

Functions that have one range value for each domain value and one domain value for each range value {bijective} can exchange domain and range {inverse function}. Reciprocals of relations can be relations {inverse relation}: x * R^1 * y = y * R^-1 * x. In the plane, inverse-function graphs reflect function graphs around line at 45-degree angle to x-axis and y-axis.

multiple-valued function

At points, inverses of single-valued functions can have multiple values {multiple-valued function} {multivalued function}. Multiple-valued functions are not functions.

postage stamp function

Functions {postage stamp function} can have intervals with higher or lower constant range. Graphs look like stair steps.

prime function

Functions {prime function} can be a sum from k = 1 to k = s of k^(-s), where p is prime number, and s is positive number. It equals product from p = 1 to p = s of (1 - p^(-s))^(-1).

reciprocal

Number or expression can have inverse {reciprocal}: 1/x.

MATH>Algebra>Function>Kinds>Bessel

Bessel function

Functions {Bessel function} can have values P0(x) = 1, P1(x) = x, ..., PN(x) = ?, where P is function over interval, N is positive integer representing interval, and x is domain value. Bessel functions include beta functions.

beta function

Gamma of z plus w can divide into gamma of z times gamma of w {beta function}: beta(w,z) = (gamma(z) * gamma(w)) / gamma(z + w). Beta functions are Bessel functions. Beta of z and w equals beta of w and z. Beta function equals integral from x = 0 to x = 1 of x^(z - 1) * (1 - x)^(w - 1) * dx.

MATH>Algebra>Function>Kinds>Complex

Cauchy formula

Holomorphic-function value at origin {Cauchy formula} is (1 / (2 * pi * i)) * contour integral (f(z)/z) * dz. Holomorphic-function value at point p is (1 / (2 * pi * i)) * contour integral (f(z)/(z - p)) * dz. Holomorphic-function nth-derivative value at origin is (n! / (2 * pi * i)) * contour integral (f(z)/z^(n + 1)) * dz.

complex function

Functions {complex function} {complex-valued function} can use complex variables: F(z), where z is complex variable. Complex function has form M + i*N, where M and N are real expressions, and i is imaginary number. For complex function F(z) = M + i*N, F(z') = M - i*N, where z' is z complex conjugate {fundamental theorem of complex numbers}.

elliptic function

Complex-number analytic functions {elliptic function} can be doubly periodic, be single-valued, have poles, and have singularity at infinity. Elliptic functions are power-series-equation solutions. They are elliptic-integral inverses. They are complex functions, because period ratios cannot be real numbers.

Weierstrass

Differential equations can have form dx^2 / dt^2 = A + B * x + C * x^2, where x is complex number. It has integral solutions {Weierstrass elliptic function}.

Jacobi

Differential equations can have form dx^2 / dt^2 = A + B * x + C * x^2 + D * x^3, where x is complex number. It has integral solutions {Jacobi elliptic function}.

Elliptic functions {Jacobian functions} can have higher powers. Determinants give solutions.

Darboux

Integrals {elliptic integral} {Darboux integral} can be (integral of P(x)) / (R(x))^0.5, where P is rational function, R is fourth-degree polynomial, and x is complex variable.

Abelian

Elliptic functions {Abelian integral} can have integral additions. They can also have algebraic and logarithmic terms. Integral of (R(u, z)) * dz, where f(u, z) = 0, requires more than one integral to describe domain. For Abelian integrals, equation genus is number of integrals needed to express solution {Abel's theorem}.

holomorphic function

Complex analytic functions {holomorphic function} {holomorphic map} {differential function} {complex differentiable function} {regular function} can be differentiable for all derivatives.

equations

Complex-function real-part derivative with variable real part can equal complex-function imaginary-part derivative with variable imaginary part. Complex-function real-part derivative with variable imaginary part can equal negative of complex-function imaginary-part derivative with variable real part {Cauchy-Riemann equations, holomorphic function}. If Cauchy-Riemann equations hold, integrating on any path between two real complex-plane points obtains same result.

holomorphy

Paths can be deformable into each other or not {holomorphy}. Deformability allows canceling by going over same portion in opposite directions {homology, holomorphy}. Non-deformability does not allow canceling {homotopy, holomorphy}. If path has singularity, different paths are not homologous. At infinitesimal limit, holomorphic functions can be conformal, non-reflective, and orientation-preserving, so the only transformations are additions and multiplications. Reciprocal functions are holomorphic. Transformations (a*z + b)/(c*z + d) {bilinear transformation} {Möbius transformation, holomorphy} are holomorphic. Laplace equation in two dimensions has holomorphic function solutions.

hyperfunction

Difference between holomorphic-function positive-frequency part and negative of negative-frequency part makes a function {hyperfunction}. Hyperfunctions have sums, derivatives, and products with analytic functions. Two hyperfunctions have no product. Hyperfunctions can represent Heaviside step function, Dirac delta function, and all analytic and holomorphic functions.

Riemann surface

Multiple-valued complex functions {Riemann surface} can be complex-plane spirals {winding space, spiral}.

point

Riemann surfaces have a central point {branch point} about which to turn.

infinity

Surfaces can rejoin after a finite number of turns {finite order, surface} or can be infinite.

logarithm function

Logarithm-function Riemann surface is not compact but can compact to Riemann sphere.

Riemann sphere

The simplest compact/closed Riemann surface {Riemann sphere} has complex plane goes through equator. Complex plane stereographically projects onto one hemisphere, and its reciprocal projects onto other hemisphere. Circles or straight lines on complex planes are circles on spheres.

genus

Sphere has genus 0, because it has no complex moduli and has three holomorphic self-transformation parameters, for bilinear transformations. Torus has genus 1, because it has one complex modulus and one holomorphic self-transformation parameter, for translation. Genus 2 has three complex moduli and no holomorphic self-transformation parameters. Genus n has 3*n - 3 complex moduli and no holomorphic self-transformation parameters.

Riemann zeta function

If Riemann hypothesis is true, functions {Riemann zeta function} can find number of primes less than N. Riemann zeta function is Dirichlet series. For complex numbers, zeta(z) = 1^-z + 2^-z + 3^-z + ..., which converges if z real part > 1, zeta(z) = 0, and z = -2, -4, -6, ... If imaginary numbers are input to zeta function, output can equal 0. Riemann zeta function equals infinity if z = 0 or 1.

Riemann hypothesis

Riemann zeta function converges if z real part = -0.5 {Riemann hypothesis} {Riemann problem}. This has no proof yet.

primes

If Riemann hypothesis is true, equation-zero locations give prime-number locations.

properties

For numbers x and N, zeta(x) = 1 + 1/2^x + 1/3^x + ... + 1/N^x. If x = 1, zeta is harmonic series. For x = 2, zeta converges to (pi)^2/6 [Euler, 1748], so sum of rational fractions gives transcendental number.

theta function

Elliptic functions can have inverses {theta function, elliptic}. Sum from z = -infinity to z = +infinity of e^(-t * n^2 + 2 * n * i * z), where n is 0 or positive integer, and t is parameter. If t > 0, theta has real part.

winding space

Multiple-valued complex functions can be sheets or spirals {winding space, function} in complex-plane Riemann surfaces.

MATH>Algebra>Function>Kinds>Continuity

continuous function

Over an interval, at domain points, function {continuous function} limits can equal range values. All polynomials are continuous functions. At points, if two functions are continuous, their sums, products, and quotients are continuous.

discrete function

Over an interval, at domain points, function {discrete function} limits can not equal range values. Functions can have values only at a finite number of points. Finite strings can represent finite discrete point sets, so discrete functions can map finite strings onto finite strings.

MATH>Algebra>Function>Kinds>Curves

catenary

Flexible heavy cable hanging from two ends makes a curve {catenary} with function y = a * cosh(x/a). y = a if x = 0.

lacunary

Functions {lacunary} can have natural boundaries.

MATH>Algebra>Function>Kinds>Exponential

error function

Functions {error function} can have value (2 / (pi^0.5)) * integral from t = 0 to t = z of e^(-t^2) * dt.

exponential function

Functions {exponential function, power} can have constant base {radix, base} {base, radix} raised to variable power: 10^x and e^x. exp(x) equals e^x if base is e or equals 10^x if base is 10. Exponential functions are logarithmic-function inverses.

Laplace transform

Functions {Laplace transform} can be integral from t = 0 to t = +infinity of e^(-s*t) * F(t) * dt. If function value is 1/s, then F(t) = 1. Integral from x = 0 to x = infinity of integral of y = 0 to y = infinity of e^(-u*x - v*y) * F(x,y) * dx * dy is Laplace transform.

logarithmic function

Functions {logarithmic function, exponential} can use variable powers of constant bases. ln(x) depends on base e {natural logarithm, base}. log(x) depends on base 10 {common logarithm}. Logarithms are exponential inverses.

Taking logarithm of value gives exponent to use with base. log(100) = x = log(10^x) = 2. ln(e^x) = x.

product

Product logarithm equals sum of factor logarithms {law of exponents}: log(M * N) = log(M) + log(N).

Exponential-function product equals exponential function of exponent sum: exp(x) * exp(y) = exp(x + y).

hyperbola

Logarithm equals area under hyperbola integrated from 1 to y, because hyperbola has equation y = 1/x, and integral of 1/y is logarithm.

line

Exponential equals 1 + n + n^2 / 2, for value n. Therefore, exponential equals 1 plus value plus area under line plus area under line at 45-degree angle.

MATH>Algebra>Function>Kinds>Filtering

comb function

Filtering functions {comb function} {shah function} {sampling function} can have unit amplitude, zero width, and unit area at regular intervals: _|_|_|_|_. Comb filter selects points or intervals.

Dirac delta function

Filtering functions {Dirac delta function} {pulse filter} can have infinite amplitude, zero width, and unit area: __|__. The pulse filter selects point or interval.

filter function

Functions {filter function} can sample another function at intervals, to measure spatial wave-front spectrum or reveal pattern shape and size.

pi function

Filtering functions {rectangle function} {pi function} can equal one over interval and equal zero elsewhere: __-__. Filter selects point or interval.

triangle function

Filtering functions {triangle function} can equal zero except over interval where it rises then falls linearly at 45-degree angle: __A__. Filter selects interval and weights middle most and ends least.

MATH>Algebra>Function>Kinds>Gamma

gamma function

Functions {gamma function} can be gamma(n) = (n! * n^z) / (z * (z + 1) * (z + 2) * ... * (z + n)), where n is number. It has limit at infinity. Gamma function is integral {Euler's integral} from t = 0 to t = infinity of t^(z - 1) * e^(-t) * dt. Value at z + 1 is z! = product from i = 1 to i = z of integral from x = 0 to x = 1 of (-log(x))^z * dx.

psi function

Functions {psi function} {digamma function} can be derivatives of gamma divided by gamma, so value at z + 1 is value at z plus 1 divided by z: psi(z + 1) = psi(z) + 1/z.

MATH>Algebra>Function>Kinds>Symmetry

even function

Functions {even function} can not change sign or absolute value when independent-variable sign changes. Even-function graphs are symmetric to y-axis.

odd function

Functions {odd function} can change sign when independent-variable sign changes. Odd-function graphs are symmetric to origin.

MATH>Algebra>Function>Kinds>Polynomial

form for polynomial

Forms {form, polynomial} are polynomial expressions. Forms can have any variable degree or number {module, polynomial} {modular system}. Forms have finite numbers of basic forms {basis, polynomial}, which make complete systems {Hilbert's basis theorem, form}.

degree

In polynomials, variable has highest exponent {degree, polynomial} {index, polynomial}. For example, polynomial x^2 + 4*x has degree 2.

graphing of function

For one-variable functions, variable power determines graph {graphing}.

Linear function has first power and is straight line, with slope and y-intercept.

Quadratic function has second power and is parabola, with two x-intercepts and one y-intercept.

Cubic equation has third power and has S shape, with three x-intercepts and one y-intercept.

Products of independent and dependent variables, x*y, are hyperbolas.

conics

General functions can add two variables, each raised to second and first power: A * x^2 + B * y^2 + C * x + D * y + E. Functions can equal zero. Equations are ellipses if b^2 - 4*a*c < 0, hyperbolas if b^2 - 4*a*c > 0, and parabolas if b^2 - 4*a*c = 0.

undetermined coefficients

If two polynomials are equal, then coefficients of terms with same variables with same exponents are equal {undetermined coefficients principle} {principle of undetermined coefficients}.

MATH>Algebra>Function>Kinds>Polynomial>Operations

polynomial operations

To add polynomials {polynomial addition}, first put terms in simplest form. Then add coefficients of terms that have same variables with same exponents. Sums have same number of terms as total number of different terms in both polynomials.

polynomial division

To divide two polynomials, write polynomials with terms decreasing from highest exponent term. Divide smaller second-polynomial first term into larger first-polynomial first term. Multiply smaller polynomial by quotient. Subtract product from first polynomial. Divide difference by second-polynomial first term, to get new quotient. Then repeat steps. For example, (12*x^2 - x - 6) / (3*x + 2) = (4*x) * (3*x + 2) - 9*x - 6 = (4*x - 3) * (3*x + 2).

polynomial multiplication

To multiply polynomials, multiply each first-polynomial term by each second-polynomial term. Product-term number is number of first-polynomial terms times number of second-polynomial terms. Put terms in simplest form. Add coefficients of terms that have same variables with same exponents.

Cartesian product

Products can result in ordered pairs {Cartesian product, function}, denoted [X,Y].

factor

A number or polynomial {factor, polynomial} can divide into another number or polynomial with no remainder. Try to find prime number that factors, try coefficient, try variable, and then try simple polynomial. Different terms can share a prime-factor product {greatest common factor, polynomial}, which divides into terms with no fractional remainder. Linear or quadratic polynomial with real coefficients can have factors with real coefficients.

factoring polynomials

Polynomials can equal smaller-polynomial products {factoring, polynomial}. For example, a^3 - b^3 factors to (a - b)*(a^2 + a*b + b^2).

difference of squares

Binomials can factor if it they are differences between two squares. For example, x^2 - y^2 factors as (x + y)*(x - y). 9*(x^2) - 64*(y^4) factors as (3*x + 8*(y^2))*(3x - 8*(y^2)).

process

To factor polynomial, first try to find number, coefficient, or variable {monomial factor} that is in all terms. Then try factor with two terms {binomial factor}. First, try binomial whose first term has coefficient that factors highest-power-term coefficient and has highest-power-term variable with no exponent. Second term is number that factors polynomial number term.

process: quadratic trinomial

To factor quadratic trinomials, first place terms in decreasing order of powers. Factor trinomial by highest-power-term coefficient. Try to factor trinomial by variable. Find constant-term numerator and denominator factors. From factors, use two numbers that add to middle-term coefficient. Then factors are (x + number1) and (x + number2).

a*(x^2) + b*x + c factors to a*(x^2 + x*(b/a) + c/a) which factors to (x + c1/a1)*(x + c2/a2), where c = c1*c2, a = a1*a2, b/a = (c1/a1 + c2/a2), and b = c1*a2 + c2*a1.

process: quadrinomial

To factor quadrinomials, first try to find a monomial factor using any term pair. For example, a + b + c + d can factor to e*(f + g) + c + d. Then try to find binomial factor shared by two term pairs. For example, 6*a*x - 2*b - 3*a + 4*b*x factors to 3*a*(2*x - 1) + 2*b*(2*x - 1) which factors to (2*x - 1)*(3*a + 2*b).

process: test binomial

If polynomial has no factors, use test binomial factor. The variable is in highest polynomial term with no exponent. Add constant. For example, x^2 + x + 1 has test factor (x + 1). Divide polynomial by test factor {synthetic division}, to get quotient polynomial and remainder polynomial {remainder theorem}. For example, (x^2 + x + 1)/(x + 1) = x + 1/(x^2 + x + 1).

If remainder is zero, test factor is polynomial factor {factor theorem}. If remainder is zero, negative of constant is a polynomial zero {converse, factor theorem}. For example, (x^2 + 2x + 1)/(x + 1) = x + 1, so remainder is zero, and x is -1.

law of quadratic reciprocity

After dividing power functions or polynomials by modulus, if remainders are the same, the power functions or polynomials are congruent quadratics {quadratic reciprocity law} {law of quadratic reciprocity}, biquadratics {law of biquadratic reciprocity}, and cubics {cubic reciprocity law} {law of cubic reciprocity}.

MATH>Algebra>Function>Kinds>Polynomial>Kinds

homogeneous function

Functions {homogeneous function} can allow factoring a constant: (k^n) * f(x, y, ...) = f(k*x, k*y, ...), where n is degree, k is constant, f is homogeneous function, and x, y, ... are variables.

linear function

Functions {linear function, polynomial} can relate variables with first power.

power function

Variable base can have constant power in function {power function}|. For example, x^3.

Clebsch-Gordan theorem

Finite complete systems for any-degree binary forms can have rational integral invariants and covariants {Clebsch-Gordan theorem}. Covariants are function projections in binary form.

Hilbert basis theorem

Forms can have any variable degree or number. Forms can use finite numbers of basic forms, which make complete systems {Hilbert's basis theorem, polynomial} {Hilbert basis theorem, polynomial}.

homogeneous expression

For all terms, sums of variable exponents can be constant {homogeneous expression}. An example is a^2 * b^1 + a^1 * b^2.

integral expression

If denominators have no variables, all variable exponents are positive {integral expression}.

perfect power polynomial

Polynomials {perfect power, polynomial} can be nth powers of similar polynomials.

perfect trinomial square

Binomials squared make trinomials {perfect trinomial square}.

quantic

Algebraic polynomials {quantic} can have two or more variables, be homogeneous, and be rational integral functions. Quantics can have two variables {binary polynomial}, three variables {ternary polynomial}, four variables {quaternary polynomial}, two degrees {quadric polynomial}, three degrees {cubic polynomial}, four degrees {quartic polynomial}, or five orders or degrees {quintic polynomial}.

radical expression

Expressions {radical expression} can contain radical signs.

rational expression

Expressions can have no radical expressions or fractional exponents {rational expression}. Rational expressions can be quotients of two polynomials. Polynomials are rational integral expressions.

MATH>Algebra>Function>Kinds>Trigonometric

trigonometric function

Trigonometry is about ratios of right-triangle sides and acute angles {trigonometric function, mathematics}.

triangle sides

Right triangle has longest side opposite right angle {hypotenuse, right triangle}, side opposite acute angle {opposite side, right triangle}, and side adjacent to acute angle {adjacent side}.

ratios

Acute right-triangle angles have ratios of opposite side to hypotenuse {sine}, adjacent side to hypotenuse {cosine}, opposite side to adjacent side {tangent, angle}, adjacent side to opposite side {cotangent}, hypotenuse to opposite side {cosecant}, and hypotenuse to adjacent side {secant, trigonometry}.

sin = opposite/hypotenuse. cos = adjacent/hypotenuse. tan = opposite/adjacent. csc = hypotenuse/opposite. sec = hypotenuse/adjacent. cot = adjacent/opposite.

trigonometric relations

Tangent equals sine divided by cosine: tan = sin/cos. Cotangent equals cosine divided by sine: cot = cos/sin.

Sine equals cosecant reciprocal: sin = 1/csc. Cosine equals secant reciprocal: cos = 1/sec. Tangent equals cotangent reciprocal: tan = 1/cot. Cotangent equals tangent reciprocal: cot = 1/tan. Secant equals cosine reciprocal: sec = 1/cos. Cosecant equals sine reciprocal: csc = 1/sin.

domain and range

For all trigonometric functions, domain is all real numbers.

The sine and cosine range is from negative one to positive one. The secant range is from positive one to infinity. Cosecant range is from negative one to negative infinity. Tangent and cotangent range is all real numbers.

angles

Trigonometric functions can have angles of less than 0 or more than 90 degrees. Trigonometric functions can have angles between 270 and 360 degrees and negative acute angles. sin(A) = -sin(360 - A). tan(A) = -tan(360 - A). csc(A) = -csc(360 - A). cos(A) = cos(360 - A). cot(A) = -cot(360 - A). sec(A) = sec(360 - A). Trigonometric functions can have angles between 180 and 270 degrees and negative obtuse angles. sin(A) = -sin(A - 180). tan(A) = tan(A - 180). csc(A) = -csc(A - 180). cos(A) = -cos(A - 180). cot(A) = cot(A - 180). sec(A) = -sec(A - 180). Trigonometric functions can have obtuse angles between 90 and 180 degrees. sin(A) = sin(180 - A). tan(A) = tan(180 - A). csc(A) = csc(180 - A). cos(A) = -cos(180 - A). cot(A) = -cot(180 - A). sec(A) = -sec(180 - A).

angles: radians

Angle 360 degrees = 2*pi radians.

angles: phase

To make angle be from 0 to 360 degrees, add or subtract multiple of 360 degrees = 2 * pi radians. All trigonometric functions repeat values for angle plus 2 * n * pi and angle minus 2 * n * pi, where n is integer. For example, sin(A) = sin(A + 2 * n * pi) and sin(A) = sin(A - 2 * n * pi).

angles: differences

Trigonometric angle-difference functions relate to trigonometric angle functions. sin(A - B) = sin(A) * cos(B) - cos(A) * sin(B). cos(A - B) = cos(A) * cos(B) + sin(A) * sin(B).

angles: negative

Trigonometric negative-angle functions relate to trigonometric positive-angle functions. sin(-A) = -sin(A). cos(-A) = cos(A). tan(-A) = -tan(A).

angles: sums

Trigonometric angle-sum functions relate to trigonometric angle functions. sin(A + B) = sin(A) * cos(B) + cos(A) * sin(B), so sin(2*A) = 2 * sin(A) * cos(A). cos(A + B) = cos(A) * cos(B) - sin(A) * sin(B), so cos(2*A) = (cos(A))^2 - (sin(A))^2 = 2 * (cos(A))^2 - 1. tan(2*A) = (2 * tan(A)) / (1 - (tan(A))^2). tan(A) = (1 - cos(2*A)) / sin(2*A) = sin(2*A) / (1 + cos(2*A)).

sums and products

Sums of trigonometric functions relate to products of trigonometric functions. sin(A + B) + sin(A - B) = 2 * sin(A) * cos(B). sin(A + B) - sin(A - B) = 2 * cos(A) * sin(B). cos(A + B) + cos(A - B) = 2 * cos(A) * cos(B). cos(A - B) - cos(A + B) = 2 * sin(A) * sin(B). Set A = (x + y) / 2 and B = (x - y) / 2 to solve for sin(x) + sin(y), sin(x) - sin(y), cos(x) + cos(y), or cos(y) - cos(x).

circular function

Trigonometric functions relate to unit circle {circular function}.

excosecant

Cosecant minus one is trigonometric function {excosecant} (excsc).

exsecant

Secant minus one is trigonometric function {exsecant} (exsec).

hyperbolic function

Functions {hyperbolic function, trigonometry} can relate to unit hyperbola as trigonometric functions relate to unit circle. For independent variable x and base e of natural logarithms, (e^x - e^-x) / 2 {hyperbolic sine} (sinh). (e^x + e^-x) / 2 {hyperbolic cosine} (cosh). (e^x - e^-x) / (e^x + e^-x) {hyperbolic tangent} (tanh). (e^x + e^-x) / (e^x - e^-x) {hyperbolic cotangent} (coth). 2 / (e^x + e^-x) {hyperbolic secant} (sech). 2 / (e^x - e^-x) {hyperbolic cosecant} (csch).

relations

(sinh(x))^2 + (cosh(x))^2 = cosh(2*x). (cosh(x))^2 - (sinh(x))^2 = 1. sinh(x + y) = sinh(x) * cosh(y) + cosh(x) * sinh(y). cosh(x + y) = cosh(x) * cosh(y) + sinh(x) * sinh(y). arcsinh(x) = ln(x + (x^2 + 1)^0.5).

principal branch function

Trigonometric functions {principal branch function} can have domain 0 to 90 degrees, for acute angles.

trigonometric inverse

If angle is acute, between 0 and 90 degrees, trigonometric functions have inverses {trigonometric inverse}: sin^-1 {arcsine}, cos^-1 {arccosine}, tan^-1 {arctangent}, cot^-1 {arccotangent}, sec^-1 {arcsecant}, and csc^-1 {arccosecant}.

versed sine

One minus cosine is function {versed sine} {versine}. One minus the sine is function {coversed sine} {versed cosine} {coversine}.

wave function

Functions {wave function} can be waves.

series

Periodic functions can be trigonometric series. If period is T, series is: a0 + sum over i of (ai * cos(2n * pi * tau / T) + bi * sin(2n * pi * tau / T)), where a0 = (1/T) * (integral from -T/2 to T/2 of f(t) * dt), ai = (2/T) * (integral of f(t) * cos(2n * pi * t / T)), and bi = (2/T) * (integral of f(t) * sin(2n * pi * t / T)).

Sine or cosine can be zero. Even periodic function uses cosine. Odd periodic function uses sine.

period

For function over interval with width x, period T is twice interval length x: T = 2*x.

jump

Term coefficients depend on differences {jump} between left-hand and right-hand function limits, derivatives at jump points, and second derivatives at jump points.

analyzer

Harmonic analyzer can find first 20 coefficients from function graph and areas. Integrator circuits can calculate area.

Fourier transform polynomial

Fourier trigonometric series {Fourier transform, function} can represent function over interval.

transformation

Function can transform by multiplying function by periodic function and integrating. Integral from x = -infinity to x = +infinity of g(x) * e^(-i * 2 * pi * u * x) * dx, where x = domain value, g(x) is function, u is frequency, and i is square root of -1.

transformation: coordinates

Fourier trigonometric series can transform coordinates. (1/(2 * pi)) * (integral from q = -infinity to q = +infinity of (e^(i*q*x)) * dq) * (integral from a = -infinity to a = +infinity of F(a) * (e^(-i*q*x)) * da).

limit

Fourier series go to limit as period approaches infinity.

time

Time t relates to phase A: t = tan(A/2).

power series

Complex power series can represent periodic functions with holomorphic positive and negative frequency.

harmonic analyzer

Mechanisms {harmonic analyzer} can find first 20 Fourier-transform coefficients, using function graphs and measuring areas.

Fresnel integral

Functions {Fresnel integral} can have value C(z) = integral, from t = 0 to t = z, of cos(pi * t^2 / 2) * dt. It equals 0.5 if z equals infinity.

Gudermannian

Functions {Gudermannian function} Gdx can have sin(u) = tanh(x), cos(u) = sech(x), or tan(u) = sinh(x) (Christoph Gudermann) [1798 to 1852].

haversine

One-half versed sine is a function {haversine} (hav).

Legendre function

Functions {Legendre function} S(z) can be integrals from t = 0 to t = z of sin((pi * t^2)/2) * dt and equal 0.5 if z equals infinity.

MATH>Algebra>Trigonometry

trigonometry

Right-triangle sides have ratios {trigonometry}.

cosine rule

In triangles, length of side c opposite angle C relates to other-two side lengths a b {law of cosines} {cosine law} {cosine rule}: c^2 = a^2 + b^2 - 2 * a * b * cos(C). If opposite angle is right angle, making right triangle, cos(C) = 0 and c^2 = a^2 + b^2, the Pythagorean theorem.

sine rule

In triangles, ratio of angle A sine to opposite-side a length is equal for all three sides {law of sines} {sine law} {sine rule} {sine formula}: sin(A) / a = sin(B) / b = sin(C) / c.

tangent law

In triangle, (a - b) / (a + b) = tan((A - B)^0.5) / tan((A + B)^0.5) {tangent law}, where angles are A and B and opposite sides are a and b.

MATH>Algebra>Problem Solving

problem solving algebra

Solving problems {problem solving, algebra} requires understanding problem or question, realizing what you know already, and knowing answer type.

hypothesis

Solving formal problem requires testing hypothetical solution {hypothesis}.

assumptions

Problem has problem context. Solving problem requires using correct assumptions about context.

principles

Believe solution is possible. Do not feel pressured, confused, or anxious. Do not think about problem difficulty or time.

Estimate and approximate, before doing details. Always try something, do not just think. Talk while doing problems to aid thinking.

If solution fails, repeat procedure to check for errors and do not become frustrated or bored.

skill

Problem-solving skill involves ability to find rules, structures, or patterns that link known with unknown.

methods

Problem solving methods are similar in all cultures, though problem types differ across cultures. People try all known methods to see if one works.

Problem-solving methods include modeling, dimensional analysis, symmetries, physical-quantity analytic properties such as differentiable or power series, parametric methods such as perturbation theory, Scene Analysis, image filtering, contour smoothing, skeletonization, polar mapping, and structural descriptions.

example

Starting at 6 PM, car 1 goes east at 100 km/hr from X toward Y. Starting at 10 PM, car 2 goes west at 80 km/hr from Y toward X. X is 800 km from Y. X is 1000 km from Z. When will the cars meet? First, read problem and make sketch with X on left and Y on right, 800 km apart, with no Z. Then realize that times, t1 = 6 and t2 = 10, speeds, v1 = 100 and v2 = 80, and distance, s = 800, have values. Remember relation between time, speed, and distance, s = v*t, where time is interval, so s1 = v1*(t12 - t1) and s2 = v2*(t12 - t2). Then use the rule that whole equals sum of its parts, to realize that s1 + s2 = 800. Solve by substitution and algebra. Realize that it needs clock time, t12 = ?, not time interval. Check dimensions, logic, and size. Reflect on method.

steps

Solving problems requires steps, from known to unknown, with reasons or examples. Verify and correct step before going to next step.

steps: 1

Specify goal and answer-type output. Classify problem. Understand problem. Read whole problem. Visualize situation, draw picture or graph, or make concrete example. Write known information. Write variable for unknown information and note which variable type it is, such as measurement, number, word, or sentence. Work on only part of large problems.

steps: 2

Gather information and connect data. Specify assumptions. Gather information and connect data. Categorize problem. Remember previous or alternative solutions. Remember equation, relation, or definition between stated variables. Look for symmetries, analogies, and simplifiers. Use thought rules and logical relations. Remember related definitions, assumptions, concepts, data, history, and causes. Look for redundant data and for insufficient data.

One rule is whole equals sum of its parts.

steps: 3

Use input and output properties to find operations or transformations necessary to derive output from input. Perform analysis to get answer. Solve problem using solution type, relation, or rule found in step two. Use reasoning, insight, or trial and error. Use conclusion drawn from data. Think "if A then B" and "B", then A is probably true {heuristic reasoning, problem solving}. Make hypothesis and try it. Do overall and most important problem part first. Do problem step-by-step, properly and neatly. Check steps immediately. Put in numbers or details after feeling solution will work. Master manipulating, rearranging, substituting, using logic, and recalling facts, to solve quickly and accurately.

steps: 4

Evaluate solution and check result. Check answer against expected answer type. Check physical dimensions. Check answer magnitude. Check against real-world knowledge. Check details for accuracy. Check logic for accuracy. Test solution in problem.

steps: 5

Think about work. Try to find shorter solution path. Remember similar problems. Remember method steps. Classify problem. Think about what to do with knowledge gained. Note other solution effects.

MATH>Algebra>Problem Solving>Methods

dimensional analysis

Measurement units can transform into equivalent measurement units with more meaning {dimensional analysis}|, to give insight into problem. Constraints on units indicate answer type needed. Solution must have correct units.

generate and test method

Problem-solving methods {generate and test method}| {trial and error} can generate answers from a possible-answer space and test whether answer solves problem.

incubation

After studying and understanding problems, solvers can do something non-intellectual {incubation}. If solutions come to mind, they must be ready to recognize solution.

MATH>Algebra>Problem Solving>Problem Types

word problem

Algebra problems {word problem} can find unknown value from conditions, using definition, theorem, or fact.

age problem

a1 = f(a2) and a2 = g(a1) {age word problem}, where a1 and a2 are ages and f and g are functions.

digit problem

d1 = f(d2) and d2 = g(d1) {digit word problem}, where d1 and d2 are digits and f and g are functions.

proportion problem

a = k(b) and f(a or b) / g(a or b) = h(a or b) / j(a or b) {proportion word problem}, where a and b are integers and f, g, h, j, and k are functions.

MATH>Algebra>Problem Solving>Problem Types>Part-Whole

fraction problem

h2 = f(h1) and 1/h1 + 1/h2 = 1 {fraction word problem}, where h1 and h2 are integers, f is a function, and 1 is equivalent to 100%.

percent problem

p1 = f(a or b), p2 = g(a or b), p1 + p2 = 1 {percent word problem}, where p1 and p2 are percentages, a and b are numbers, f and g are functions, and 1 is equivalent to 100%.

percent mixture problem

h1 = f(a or b), h2 = g(a or b), and 1/h1 + 1/h2 = 1 {percent mixture word problem}, where h1 and h2 are concentrations, a and b are numbers, f and g are functions, and 1 is equivalent to 100%.

rate of work problem

h1 = f(a or b), h2 = g(a or b), and 1/h1 + 1/h2 = 1 {rate of work word problem}, where h1 and h2 are time, a and b are numbers, f and g are functions, and 1 is equivalent to 100%.

MATH>Algebra>Problem Solving>Problem Types>Total

area problem

For triangle, A = 0.5 * l * h {area word problem}, where A is area, h is height, and l is side length. For rectangle, A = l*h, where A is area, and l h are side lengths. For circle, A = pi * r^2, where A is area, and r is radius.

interest problem

P + P * r^t = T {interest word problem} {savings word problem} {investment word problem}, where P is principal, t is time, r is rate, and T is total.

perimeter problem

For triangle, p = a + b + c {perimeter word problem}, where p is perimeter, and a b c are side lengths. For rectangle, p = a + b + c + d, where p is perimeter, and a b c d are side lengths. For circle, p = 2 * pi * r, where p is perimeter, and r is radius.

uniform motion problem

d = v*t {uniform motion word problem} {rate word problem} {time word problem} {distance word problem}, where d is distance, v is speed, and t is time. vf^2 = vi^2 + 2 * a * ds, where ds is distance, vf is final speed, vi is initial speed, and a is acceleration.

volume problem

For box, V = l*h*w {volume word problem}, where V is volume, and l h w are side lengths. For sphere, V = (4/3) * pi * r^3, where V is volume, and r is radius.